Comparison of Direct-Radiator Loudspeaker System Nominal Power Efficiency vs. True Efficiency with High-BI Drivers

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ABSTRACT

Recently Vanderkooy et al. [1, 2] considered the effect on amplifier loading of dramatically increasing the Bl force factor of a loudspeaker driver mounted in a sealed-box enclosure. They concluded that high Bl was a decided advantage in raising the overall efficiency of the amplifier-speaker combination particularly when a class-D switching-mode amplifier was used. When the Bl factor of a driver is raised dramatically, the input impedance magnitude also rises dramatically while the impedance phase essentially approaches a purely reactive condition of ±90° over a wide bandwidth centered at resonance. This is an optimum load for a class-D amplifier, they note, which not only can supply power, but can also efficiently absorb, store, and return power to the speaker. Unfortunately, the system designed with a high-BI driver requires significant low-frequency equalization and increased voltage swing from the amplifier as compared to systems using typical much-lower values of Bl. This paper considers the effect on the driver's efficiency of raising the driver's Bl factor through a series of Spice simulations. The nominal power transfer efficiency defined in traditional loudspeaker design methods is contrasted with true efficiency, i.e. true acoustic power output divided by true electrical power input. Increasing Bl dramatically increases the driver's true efficiency at all frequencies but radically decreases nominal power efficiency in the bass range. Traditional design methods based on nominal power transfer efficiency disguise the very-beneficial effects of dramatically raising the driver's Bl product.
0. INTRODUCTION
Recently Vanderkooy et al. [1, 2] wrote papers concerning high-efficiency loudspeakers where they analyzed the effect on amplifier loading of increasing the efficiency of a direct-radiator loudspeaker driver by raising its Bl product by a large amount. The driver’s Bl force factor relates the input current and the resultant force applied to the driver’s voice coil. They compared the driver-loading effect on several different types of amplifiers of two different closed-box loudspeaker systems using the same size driver: 1. a system designed using traditional design techniques with a moderate Bl that maximizes acoustic low-frequency extension and response flatness when driven by a constant voltage source, and 2. a second identical system with the same driver whose Bl factor was increased by a factor of five.

They concluded that the high-Bl factor driver provided an extremely good match to a switching-mode class-D amplifier and maximized the efficiency of the amplifier and loudspeaker in combination. They pointed out that the combination of a high-Bl driver driven by a class-D amplifier could have an overall efficiency greater than ten times that of the traditionally-designed moderate Bl system driven by a typical class-B amplifier. Stated another way, the actual power drawn by a class-D amplifier driving the high-Bl system was less than one-tenth that of a traditional system generating the same acoustic output power.

Vanderkooy et al. determined that the high-Bl factor significantly raised the input impedance of the driver and results in an impedance that is essentially reactive over a very-wide band in the operating range of the driver with a phase angle that approaches ±90°. They concluded that this reactive load was an optimum match to a class-D switching-mode amplifier because the amplifier could not only supply power to the load but could also absorb reactive load energy and return it to the amplifier’s power supply, thus increasing total efficiency. As they also point out, the downside of increasing the Bl product is the requirement that the amplifier must provide greater voltage swing and significant bass equalization is required to drive the speaker to flat response as compared to the moderate-Bl driver system.

Although Vanderkooy et al. primarily emphasized the combined efficiency of the amplifier-speaker combination, I believe they somewhat downplayed the effect of high Bl on the true power efficiency of the driver itself. Raising the Bl force factor of a driver raises the true efficiency (ratio of acoustic power output to actual electrical power input) at all frequencies, but severely attenuates the bass response as defined by nominal power efficiency. The traditional design techniques based on nominal power efficiency thus effectively disguises the true effects of raising the Bl product of the driver, and strongly discourage designers from choosing higher Bl factors because of the perceived detrimental effect on bass response.

This paper illustrates the effects of raising the Bl product of a driver by a series of Spice circuit analysis simulations. The same 8”-driver system modeled by Vanderkooy et al. is simulated here with the same Bl factor jump from 8 to 40 N/A, a factor of 5 increase. The Spice simulations are used to illustrate the effects of raising the Bl factor on the driver’s input impedance, nominal power transfer efficiency, and on its true power transfer efficiency. The Spice circuits include the air radiation load impedance (both real and imaginary parts) of the 8” driver and thus are a more accurate model of the driver’s predicted efficiency as compared to the model used by Vanderkooy and Boers.

This paper is organized as follows. Section 1 describes the definition and assumptions of the efficiency definition used by the traditional design methods, based on nominal electrical input power. Section 2 describes true power transfer efficiency which is based on the actual electrical input power and the radiated acoustic output power. Section 3 describes the effects on the traditional design of dramatically raising the Bl factor. Section 4 describes the Spice circuit closed-box speaker system models that generated the data for the comparison of results for the two Bl conditions. Section 5 compares the low Bl and high Bl design’s input impedance and efficiency frequency responses for the two efficiency definitions. Section 6 concludes and section 7 lists the references. The appendix describes the electrical equivalent circuit used for the driver in its closed-box enclosure (including air radiation load) along with the definition of the circuit values, their equations, and defines other symbols used in this paper.

1. NOMINAL POWER TRANSFER EFFICIENCY

Conventional loudspeaker low-frequency design techniques optimize the design for constant-voltage operation according to the teachings of Thiele and
Small [3-5]. This is as it should be, because speakers are ordinarily driven by amplifiers with very-low output impedances which provide essentially constant-voltage operation regardless of loudspeaker impedance. Speakers are also traditionally designed to have roughly flat acoustic frequency response when presented with a constant-voltage flat-response electrical input.

These operating conditions and assumptions drove the design techniques and particularly the definition of the electro-acoustic efficiency of a speaker system, the so-called nominal power transfer efficiency, which is defined as the acoustic power output divided by the nominal electrical input power.

1. Nominal Electrical Input Power

The nominal electrical input power to a loudspeaker driver or system is defined as the power delivered by the amplifier into a resistor having the same value as the driver’s voice coil resistance (or sometimes defined as the driver’s rated impedance or minimum impedance in the system’s pass band). This is usually calculated by simply squaring the input voltage and dividing by the driver’s voice coil resistance or rated impedance. This definition of input power yields an efficiency vs. frequency response curve that mimics the SPL response curve you get when driving the system with a constant voltage source.

2. True Power Transfer Efficiency

Unfortunately the nominal power transfer definition of efficiency completely disguises what happens to the actual or true efficiency of the driver when its Bl product is changed. The true efficiency of the driver is defined as the acoustic power output divided by the true electrical input power.

3. Effect of Raising Bl on the Traditional Design

Vanderkooy et al. [1, 2] analyze a traditional model of a loudspeaker mounted in a closed-box enclosure which follows the general modeling techniques and assumptions of Thiele and Small [3-5]. They analyze the effects of raising the Bl force factor from the traditional design value of 8.0 N/A to 40 N/A, an increase of 5 times.

3.1. Traditional Model Assumptions and Problems

These models are based on acoustical analogous circuits that are valid only for frequencies within the piston range of the driver, i.e. for low frequencies where the radiated wavelengths are larger than the circumference of the driver (\(ka<1\), where \(a\) = radius of the driver and \(k\) is the wave number). Circuit elements which do not contribute enough impedance to affect the analysis, such as radiation loads, are also neglected. Voice coil inductance is also not considered.

Because the traditional model neglects air radiation loads (however, the radiation air-load mass is included and adds to the total moving mass of the driver), the traditional loudspeaker design model of Thiele and Small potentially overestimates the efficiency of the driver because it essentially assumes that driver is very inefficient to start with [6]. The Thiele-Small model assumes that driver is strictly operated in its piston range where its dimensions are small in relation to wavelength. This is not correct at higher frequencies where the driver’s dimensions are significant with respect to wavelength. Furthermore, when the Bl force factor of a design is raised arbitrarily, the radiation load impedances are significant when compared to other model impedances.

The following simulations do include the radiation load as a part of the model. The main effect of this inclusion is to limit the maximum efficiency of the designs, particularly when the Bl product is raised to high values, and to roll off the high-frequency response.

3.2. Driver and Box Parameters

Vanderkooy et al. define a typical 8”-driver closed-box system to illustrate the effects of raising the Bl force factor on the design. When the driver is mounted in the closed-box enclosure, the driver’s resonance rises from 30 to 81 Hz. The following lists the mechanical and Thiele-Small parameters of the design for both the Bl =8 N/A and BL = 40 N/A
conditions. Refer to the Appendix for definitions of symbols.

3.2.1. Mechanical Parameters

\[ M_{MD} = 0.01 \text{ kg} \]
\[ C_{MS} = 2.8 \times 10^{-3} \]
\[ a = 0.08 \text{ m} \]
\[ Bl = 8.0 \text{ N/A or 40 N/A} \]
\[ R_E = 6.0 \text{ Ohms} \]
\[ R_{MS} = 1.0 \text{ N.s/m} \]
\[ V_B = 0.025 \text{ m}^3 \]

3.2.2. Thiele-Small Parameters

\[ f_S = 30.0 \text{ Hz} \]
\[ f_C = 81.4 \text{ Hz} \]
\[ V_{AS} = 0.159 \text{ m}^3 \]
\[ Q_{ES} = 0.177 (Bl = 8) \text{ or } 0.0071 (Bl = 40) \]
\[ Q_{MS} = 1.89 \]
\[ Q_{TS} = 0.162 (Bl = 8) \text{ or } 0.0071 (Bl = 40) \]
\[ \eta_0 = 2.4 \% \text{ (Bl = 8) or } 60 \% \text{ (Bl = 40)} \]

3.3. Effect on Efficiency Frequency Response Modeled Using Traditional Techniques

Figure 1 shows the nominal power efficiency versus frequency for the two different Bl values. The plots were generated using the assumptions of Thiele and Small outlined in section 3.1. This graph plots the same data as Fig. 3 of [1] but with a scale in dB referenced to 100 \% (0 dB = 100 \%) rather than the 1W/1m frequency response in dB SPL.

As Vanderkooy et al points out, raising the Bl factor of an optimally designed closed-box system from its designed value of 8 N/A to the much-higher value of 40 N/A, dramatically increases the predicted mid-band efficiency by 14 dB which rises from 2.4 \% to 60 \%, but features a greatly rolled-off over-damped low-frequency response with about 15 dB less bass output than the original system at frequencies at and near the original system’s 81 Hz cutoff frequency.

On first examination using traditional criteria, the high-Bl second system would be immediately dismissed because of its vastly attenuated low-frequency response. This is not the whole story however. This strong judgement for the first system and against the second is based strictly on driving the system with a constant input voltage and indirectly on the traditional assumption that the input impedance of the system is constant at a value equal to the system’s rated impedance.

Both of these latter conditions are a result of using traditional models that mandate that the frequency response be calculated using the concept of nominal power transfer efficiency (Section 1). As will be shown later, when the frequency response is based on the true power transfer efficiency definition (Section 2), the second system appears much more favorable.
When the true efficiency of the driver is considered, it is clear that increasing the Bl factor will directly result in higher efficiency values at all frequencies. Unfortunately, the constant-voltage-drive low-frequency response may suffer, but this only means equalization must be used to flatten the frequency response.

4. SPICE CIRCUIT MODELS

This section describes the Spice circuit models that generated the data for the plots of predicted efficiency vs. frequency and input impedance of the driver mounted in the closed-box enclosure for the two Bl conditions.

4.1. Simulation of Input Impedance

Figure 2 shows the Spice circuits that simulated the input impedance of the closed-box systems for the two different values of Bl. The appendix describes the electrical equivalent circuit used for the driver in its closed-box enclosure along with the definition of the circuit values, their equations, and other symbols used in this paper. All circuit models neglect voice-coil inductance. Air loads are simulated by a series resistor-capacitor (RC) high-pass network.

All the components of the model, except for the dc resistance of the voice coil, depend on Bl squared. Inductors and resistors are proportional to Bl squared, while capacitors are inversely proportional. When the Bl is raised by a factor of 5, all resistors (except for the voice-coil resistors R1 and R6) and inductors increase by a factor of 25, while all capacitors decrease by the same factor.

The input impedance magnitude and phase was calculated by simply dividing the input voltage by the input current.

4.2. Simulation of Efficiency vs. Frequency

Figure 3 displays the Spice circuits used to calculate the simulated efficiency frequency response of both Bl conditions. The power transfer efficiency was calculated by dividing the output power by the input power. Note that the input power is depends on the definition of efficiency, either nominal electrical input power or true electrical input power as described in Sections 1 and 2.
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on the bottom was used to calculate the nominal power input for the rated 6-ohm input impedance. The power output was calculated by multiplying the voltage across and current in the respective output resistors R3 and R8. The true power input was calculated by multiplying the circuits input voltage by the real part of the input current. The efficiency was calculate by dividing the output power by the input power.

5. COMPARE DESIGNS for LOW BI and HIGH BI CONDITIONS

The following sub sections describe the output of the Spice simulations comparing the two BI conditions for the closed-box loudspeaker system design. Input impedance magnitude and phase, nominal power transfer efficiency frequency response, and true power transfer efficiency frequency response were compared for the low BI and high BI conditions.

5.1. Input Impedance

5.1.1. Magnitude

The simulated input impedance magnitude of the two BI conditions are shown in Figs. 4 and 5. Figure 4 plots the data on a linear vertical scale, while Fig. 5 shows the same data on a logarithmic vertical scale. Both figures illustrate the large increase of input impedance when the BI is raised. For the high BI condition, not only has the input impedance increased dramatically at and around resonance, but also over a very-wide two-decade range centered at resonance! At resonance, the impedance has increased from 72 to 1,660 Ohms, an increase of 23 times! This increase of input impedance is the primary reason for the efficiency increase of the high-BI designs.

5.1.2. Phase

Figure 6 shows the phase of the input impedance for the two BI conditions. With high BI, the phase indicates that the loudspeaker load is essentially reactive over a very-wide range frequency range centered at resonance (also noted in [1, Fig. 2]). The phase magnitude stays at and above 75° over a two-decade range!

5.2. Compare Nominal Power Transfer Efficiency without Air Load

Figure 1, shown previously, shows the nominal power transfer efficiency frequency response
comparison for the two BI conditions for the conventional Thiele-Small model that neglects air load (except for the added mass of the air load). Note that this model predicts a dramatic rise of mid-band efficiency from 2.4 to 60% (+14 dB) and a low-frequency loss over a very-wide two-decade bandwidth extending from 8 to 800 Hz.

As pointed out before, the curve is a direct result of the Thiele-Small design assumptions that dictate a constant input voltage, a constant nominal electrical input power, and disregard the effects of the driver’s radiation air load. These assumptions disguise the very beneficial effects of raising the BI product.

In addition, the very-high predicted efficiency of 60% is immediately suspect because of its high value. Keele [6] shows that the maximum efficiency of a direct-radiator loudspeaker is limited to 25% automatically by the definition of nominal power transfer efficiency.

5.3. Compare Nominal Power Transfer Efficiency with Air Load

Figure 7 shows the nominal power transfer efficiency frequency response comparison of the two designs based on a model that includes the air load. Inclusion of the air load severely restricts the increase of efficiency with BI and causes a roll-off of efficiency at high frequencies. Maximum nominal power transfer efficiencies are 1.5% for BI = 8 N/A and 2.6% for BI = 40 N/A. Note that although the air load is included, a severe loss of low frequencies is still observed because of the way efficiency is defined. Note also that the maximum efficiency has dropped from 60% down to 2.6% when the air load radiation impedance is included.

Fig. 7. Comparison of the nominal power transfer efficiency of the systems with BI = 8 N/A and BI = 40 N/A as calculated from the Spice circuit of Fig. 3. This graph is a more accurate prediction of efficiency as compared to the data of Fig. 1 which essentially neglects the air radiation load. The main effect of the radiation load is to roll-off the efficiency above 1 kHz. Note that the high BI has only raised the predicted maximum nominal power transfer efficiency from 1.5% to 2.6%, and as before has severely attenuated the bass response.

5.4. Compare True Power Transfer Efficiency with Air Load

Figure 8 compares the true power transfer efficiency of the two designs, both with air load. Maximum true power transfer efficiencies are 5.1% for BI = 8 N/A and 25.6% for BI = 40 N/A. Here the very beneficial effects of raising the BI factor are clearly evident. The high value of BI has not only raised the true maximum efficiency by a factor of five but has also increased the efficiency over the whole operating bandwidth of the transducer. The efficiency increase approaches 14 dB or 25 times at high and low frequencies.

Fig. 8. Comparison of the true efficiency of the systems with BI = 8 N/A and BI = 40 N/A as calculated from the Spice circuit of Fig. 3. Compare this data to the previous figure (Fig. 7). The high value of BI has not only raised the true maximum efficiency by a factor of five but also has increased the efficiency over the whole operating bandwidth of the transducer.

5.5. Compare Nominal Power Efficiency and True Power Efficiency for low BI

Figure 9 compares the nominal power transfer efficiency and the true power transfer efficiency, at the low value of BI (= 8 N/A). At this low BI value, the true efficiency increases only over a relatively narrow two-octave range centered at resonance, which directly corresponds to the narrow impedance
peak over the same range for the low Bl value (Figs. 4 and 5).

5.6. **Compare Nominal Power Efficiency and True Power Efficiency for high Bl (= 40 N/A)**

Figure 10 likewise compares the nominal power transfer efficiency and the true power transfer efficiency, but for the high value of Bl (= 40 N/A). Here the comparison is highly skewed towards the high Bl condition with efficiency increases in excess of 24 dB within an octave of resonance, and extending over a very wide range with significant increases.

![Fig. 9. Comparison of the nominal power efficiency and true power efficiency for the system with low Bl (= 8 N/A) as calculated from the Spice circuit of Fig. 3.](image)

![Fig. 10. Comparison of the nominal power efficiency and true power efficiency for the system with high Bl (= 40 N/A) as calculated from the Spice circuit of Fig. 3.](image)

3. Note that for this high value of Bl, the true efficiency is very much higher than the nominal power efficiency over most the operating bandwidth of the system. At 100 Hz, the true efficiency predicts a value 8.5% which is 24 dB greater than the 0.033% value predicted by the nominal power efficiency.

6. **CONCLUSIONS**

Traditional loudspeaker design methods optimize the design to yield flat acoustic output frequency response when driven by a constant voltage source. This assumption made it convenient to define the system’s electro-acoustic conversion efficiency as the transfer ratio between to the nominal electrical input power and the acoustic output power of the system. This efficiency is called the nominal power transfer efficiency.

The use of the nominal electrical input power \( V_i^2 / R_e \) in the efficiency definition is convenient because it is constant with frequency and depends only on the input voltage and the dc or rated (or minimum) impedance of the system. Using this definition of input power means that the frequency response of the efficiency (the square of the system sensitivity ratio or the system frequency response) is identical to the actual frequency response of the system when driven by a constant voltage.

Designs that result from this efficiency definition are clearly optimized for constant input voltage operation, as they should be. However, as a result of this operating constraint, the Bl force factor of the design tends to a value that optimally extends the low-frequency response of the system. Higher or lower values of Bl are judged undesirable because these values cause unacceptable changes in the frequency response.

Judged in the light of traditional design methods, high-Bl designs are severely downgraded because of the severe loss of low-frequency response. If the restriction of constant voltage operation is relaxed, i.e. before-the-power-amplifier equalization is acceptable, the true efficiency of the driver can be significantly increased by raising the Bl factor. Significantly raising Bl can dramatically increase the driver’s true efficiency over a very wide band because the input impedance rises dramatically. Traditional design methods completely disguise this very beneficial effect.

The downside of increasing the Bl product is the requirement that the amplifier must provide much greater voltage swing and significant bass equalization is required to drive the speaker to flat
response as compared to the moderate-Bl driver system.

To conclude, if your design can accommodate equalization before the power amplifier and the power amplifier can provide higher voltage swing, then raise your driver’s Bl product to the highest possible value consistent with material and economic constraints! This will result in the highest efficiency design.

7. REFERENCES
APPENDIX: SYSTEM ELECTRICAL EQUIVALENT CIRCUIT

The electrical equivalent circuit of a driver mounted in a closed-box enclosure with air radiation load is shown in Fig. 11. The air load model, an RC high-pass filter, is taken from Beranek [7, p. 124] (Note: This simple RC network model and values are technically only valid for \( ka < 0.5 \), but are used here to approximate the air load over the complete frequency range).

![Electrical equivalent circuit diagram](image)

Fig. 11 Electrical equivalent circuit of a moving-coil electro-dynamic driver mounted in a closed-box enclosure with air radiation load. Driver voice-coil inductance is neglected.

Only the air load on the front of the driver is considered. Driver voice-coil inductance is neglected. In this circuit the electrical values are defined in the following list along with other symbols:

- **\( C_{AR} \)** electrical capacitance due to acoustic radiation air load mass on front of driver
  
  \[ \left( \frac{M_{AR}S_D}{B^2l^2} \right) \]
  
  \[ = \frac{8}{3} \rho \alpha^3 / B^2l^2 \]
  
  \[ = 2.67 \rho \alpha^3 / B^2l^2 \]

- **\( C_{MED} \)** electrical capacitance due to driver moving mass excluding air load
  
  \[ \left( \frac{M_{MD}S_D}{B^2l^2} \right) \]

- **\( C_{MS} \)** mechanical compliance of driver suspension

- **\( f_S \)** resonance frequency of driver

- **\( f_C \)** resonance frequency of driver mounted in closed-box enclosure

- **\( L_{AB} \)** electrical inductance due to acoustic compliance of air in enclosure
  
  \[ = \frac{C_{AB}B^2l^2}{S_D^2} \]

- **\( L_{CES} \)** electrical inductance due to driver compliance
  
  \[ = \frac{C_{AS}B^2l^2}{S_D^2} \]

- **\( M_{MD} \)** mechanical mass of driver diaphragm assembly excluding air load

- **\( \eta_0 \)** mid-band reference efficiency of driver

- **\( Q_{ES} \)** \( Q \) of driver at \( f_S \) considering electrical resistance \( R_E \) only

- **\( Q_{MS} \)** \( Q \) of driver at \( f_S \) considering driver non-electrical resistances only

- **\( Q_{TS} \)** Total \( Q \) of driver at \( f_S \) including all driver resistances

- **\( R_{AR} \)** electrical resistance due to acoustic radiation resistance

- **\( R_E \)** dc electrical resistance of driver voice coil

- **\( R_{ES} \)** electrical resistance due to driver suspension losses
  
  \[ = \frac{B^2l^2}{S_D^2R_{AS}} \]

- **\( R_{MS} \)** mechanical resistance of driver suspension losses

- **\( V_B \)** volume of enclosure

- **\( V_{AS} \)** volume of air having same acoustic compliance as driver suspension
The other parameters and physical constants appear as:

- $B$: magnetic flux density in driver air gap
- $c$: velocity of sound in air (=343 m/s)
- $l$: length of voice-coil conductor in magnetic field
- $a$: radius of driver diaphragm
- $S_d$: effective projected surface area of driver diaphragm
- $\rho_0$: density of air (= 1.21 kg/m$^3$)