A LOUDSPEAKER HORN THAT COVERS A FLAT RECTANGULAR AREA FROM AN OBLIQUE ANGLE

D. B. (Don) Keele, Jr.
JBL Incorporated
Northridge, California

Presented at the 74th Convention 1983 October 8-12 New York

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AN AUDIO ENGINEERING SOCIETY PREPRINT
A LOUDSPEAKER HORN THAT COVERS A FLAT RECTANGULAR AREA FROM AN OBLIQUE ANGLE

by

D. B. (Don) Keele, Jr.
JBL Incorporated
Northridge, CA 91329
U. S. A.

A constant-directivity defined-coverage loudspeaker horn that approximately covers a flat rectangular area from an oblique angle is described. The horn compensates roughly for the inverse rolloff of sound pressure in the forward-backward direction by varying the horizontal coverage as a function of the elevation angle. The horizontal coverage angle of the horn at the 6-db-down points at each elevation angle is matched to the required horizontal angle of the rectangular area as seen by the horn.

Measurements on a prototype horn which covers a two by three unit area from a point one unit above the center of the narrow end are shown. This horn roughly covers a 70 deg vertical angle with the horizontal coverage smoothly changing from 90 degs at 0 deg elevation (straight down) to 38 degs at 70 degs elevation.

0 INTRODUCTION

In most sound reinforcement and playback systems one of the primary goals is to attain even coverage of the direct-field sound pressure at all the listener positions. More than likely the shape and configuration of the region that you want to cover does not match the polar pattern of any single loudspeaker you may want to use. This usually leads to a multi-source loudspeaker system design composed of a number of sources each of which cover a portion of the total listener area.

In a number of situations such as churches and theaters the high acoustic output capabilities of a multi-source system is not required. In these situations the multiple sources are used only for better coverage of the listening area. Often a single source system could be used due to moderate sound level requirements but only if it has the proper directional characteristics.

This paper describes a single loudspeaker horn which provides a defined coverage pattern roughly matched to the common situation of a rectangular area as seen from an oblique angle.
1 HISTORY

In the past, solutions to typical coverage situations included: 1. Arrays of loudspeakers (Klepper [1], Schneider [2], Jones [3]), 2. Gradient sources (Hilliard [4], Olson [5]), 3. Horns (Wente [6], Klipsch [7], Keele [8], Henrikson [9]), and 4. Central clusters (Malmlund [10], Patronis [11], Rosner [12], Figwar [13]).

2 THEORY

2.1 Required Angular Coverage

Fig. 1 shows the general situation of an acoustic source above and on the center line of a rectangular region. The task is to derive the functional relationship between the required horizontal coverage angle (as seen by the source) and the elevation angle.

In Fig. 1 the source is H units above the plane of the rectangle and L1 units behind and on the center line of the rectangle. The rectangle is W units wide and L units long. The elevation angle is alpha (defined with 0 degs straight down) and the total included horizontal coverage angle is beta.

Assuming a rectangular coordinate system centered directly below the source on the plane of the rectangle with the positive x axis bisecting the rectangle, analysis of Fig. 1 yields the following relationships:

for the elevation angle

\[ \alpha = \arctan \left( \frac{x}{H} \right) \]  \hspace{1cm} (1)

where

- \( x \) is the distance from the origin to an arbitrary point on the center line of the rectangle (the x axis),
- \( H \) is the height of the source over the plane of the rectangle; and

for the horizontal coverage angle (as seen by the source)

\[ \beta = 2 \cdot \arctan \left( \frac{W}{2 \cdot \sqrt{x^2 + H^2}} \right) \]  \hspace{1cm} (2)

where

- \( W \) is the width of the rectangle.

Note: all equations in this paper are shown using standard computer language notation (BASIC) where * = multiply, / = divide, ^ = exponentiation, sqrt = squareroot function, arctan = inverse tangent function, etc.
Two sets of reference elevation and horizontal coverage angles can be defined at the front and rear of the rectangle:

at the front (x = L1)

\[ \alpha_1 = \arctan \left( \frac{L_1}{H} \right) \]

\[ \beta_1 = 2 \cdot \arctan \left( \frac{W}{2 \cdot \sqrt{L_1^2 + H^2}} \right) \]

at the rear (x = L1 + L)

\[ \alpha_2 = \arctan \left( \frac{L_1 + L}{H} \right) \], and

\[ \beta_2 = 2 \cdot \arctan \left( \frac{W}{2 \cdot \sqrt{(L_1 + L)^2 + H^2}} \right) \]

A more general analysis of a source over a plane may be found in Cable [14].

2.2 Horn Design

Once the horns horizontal coverage angle is known as a function of the elevation angle (eqs. 1, 2) it is straightforward to compute the contours of the horn sidewalls at each elevation angle. Well known sidewall contours as shown in [7], [8], [9], and Smith [15] can be used. The generated horn bell section can then be joined to a driver coupler with a throat section of exponential or other area increase function [7], [8], [9], and [15].

3 EXPERIMENT

3.1 System Parameters

A defined coverage horn was designed to cover a 2 by 2.75 (roughly 2 by 3) unit rectangle from a point one unit above the center of the narrow end. These requirements result in the following system parameters:

rectangle length = L = 2.75,
rectangle width = W = 2.0,
source height = H = 1.0, and
source offset = L1 = 0.0.

The horn was to be used with a one inch diameter throat compression driver and have an operating range of roughly 500 Hz to 12.5 kHz.
3.1 Elevation and Coverage Angles

Using eqs. (3 - 6), these system parameters result in the following elevation and horizontal coverage angles:

at the front of the rectangle ($x = 0$, source end)

\[ \text{elevation angle} = \alpha_1 = 0.0 \text{ degs (straightdown)}, \]
\[ \text{coverage angle} = \beta_1 = 90.0 \text{ degs}; \]

at the rear of the rectangle ($x = 2.75$)

\[ \text{elevation angle} = \alpha_2 = 70.0 \text{ degs}, \]
\[ \text{coverage angle} = \beta_2 = 37.7 \text{ degs}. \]

Fig. 2 shows side and perspective views of the selected system parameters.

3.3 Horn Design

3.3.1 Coverage vs Elevation Angles

The horn's required horizontal coverage at each elevation angle is computed from eqs. (1) and (2) as follows (both a function of the running parameter $x$):

\[ \text{elevation angle} = \alpha = \arctan (x), \]
\[ \text{coverage angle} = 2 \times \arctan \left( \frac{1}{2 \times \sqrt{x^2 + 1}} \right). \]

These two equations were used to generate the data shown in Table 1 which shows the horn's elevation angle and horizontal coverage angle at several $x$ axis points.

<table>
<thead>
<tr>
<th>$x$ (Normalized units)</th>
<th>Elevation Angle (Degs)</th>
<th>Coverage Angle (Degs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>90.0</td>
</tr>
<tr>
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<tr>
<td>1.73</td>
<td>60.0</td>
<td>53.1</td>
</tr>
<tr>
<td>2.14</td>
<td>65.0</td>
<td>45.8</td>
</tr>
<tr>
<td>2.75</td>
<td>70.0</td>
<td>37.7</td>
</tr>
</tbody>
</table>
3.3.2 Sidewall Contours

The sideline contour generating equation shown in [eq. p.409, 15], and Keele patent [16] was used to define the sideline contour shapes in both planes to yield the smoothest off-axis frequency response:

\[ y = a + b \cdot x + c \cdot x^n \]  

(7)

where

- \( x \) = distance from throat of horn or source aperture gap,
- \( y \) = horn contour dimension at point \( x \),
- \( a \) = constant which defines half width (or half height) of start of horn (either the throat radius or one-half the aperture gap width, this dimension sets the highest operating frequency of the horn up to which full rated beamwidth is maintained),
- \( b \) = constant which defines the slope of the sideline near the throat or aperture gap of the horn \((b = \tan(\text{desired-coverage-angle}/2))\),
- \( c \) = constant which sets the amount of end flaring of the sideline (flaring of the sideline near the mouth of the horn minimises mid-range narrowing [8], and
- \( n \) = power constant which defines the rapidity of sideline flare \((= 7 \text{ for this design}).

Eq. 7 was used to define the horizontal sideline shape of the horn at each elevation angle in a smooth progression. The horizontal mouth width of the horn was constant for all vertical elevation angles. The vertical sidewalls (0 deg and 70 deg contours) were also designed using eq. 7. The general teachings of patent [16] were used to design the rest of the horn (exponential throat section etc.).

3.3.3 Horn Drawing

The complete horn design is shown in Fig. 3 and in isometric sketch form in Fig. 4. The horn is roughly 760 mm (30 in) long, 1025 mm (40.4 in) high, and 780 mm (30.7 in) wide.

The selected mouth dimensions maintain horizontal beamwidth control down to roughly 850 Hz for the narrow-angle (38 deg) coverage long-throw part of the horn and down to 360 Hz for the wide-angle (90 deg) coverage short-throw portion of the horn.

3.4 Model Construction

The model was constructed from wood and modeling compound. The complex compound curve shapes of the horizontal walls were built up from each individual horizontal contour at each elevation angle (at roughly 10 deg angle increments) much the same way the contour of an airplane wing is built up.

Photos of the completed experimental prototype horn are shown in Fig. 5.
3.5 Model Measurements

The model was tested using a JBL model 2425 compression driver. All measurements were done over a large ground plane (flat concrete surface) at a distance of 2 meters from the mouth of the horn using the technique of Gander [17]. A fixture was constructed so that the horn could be rotated easily in the elevation plane (Fig. 5).

3.5.1 Zero Degree Elevation Frequency Response and Impedance

Fig. 6 shows the measured 0 deg elevation (90 deg horizontal coverage at this elevation angle) frequency response and impedance of the horn. This curve was run at 2 meters from the mouth with an input voltage of 2.83 Vrms (equivalent to 1 watt 1 meter in free space). Due to the constant directivity characteristics of the horn the frequency response clearly shows the power response of the compression driver. The drivers power response is roughly flat from 500 Hz to 3 kHz with a 6 dB per octave rolloff beyond.

3.5.2 Horizontal Off-Axis Response Curves

Fig. 7 shows a family of horizontal off-axis frequency response curves at several selected vertical elevation angles. At each elevation angle, 8 response curves were taken at all off-axis horizontal angles between 0 to 70 degs in steps of 10 degs. Eight complete sets of horizontal off-axis curves were gathered at all elevation angles between 0 and 70 degs in steps of 10 degs. Levels have been preserved from one set to another with the SPL indicated on the graphs.

These horizontal response curves show a well behaved off-axis behavior that progressively gets narrower and narrower as the elevation angle increases. Again the constant directivity nature of the horn shows up very clearly in the uniformly parallel off-axis curves.

3.5.3 Vertical Off-Axis Response Curves

Fig. 8 shows a composite set of off-axis frequency response curves taken at all elevation angles between 0 and 70 degs in steps of 10 degs (all at 0 deg horizontal angle). The 40 and 50 deg curves appear to exhibit the highest sensitivity.

The desired highest sensitivity at the 70 deg elevation angle is not evident. Thus this horn will only partially compensate for the inverse rolloff of sound pressure in the forward-backward direction.
3.5.4 Polar, Beamwidth, Directivity, and Isobar Measurements

Due to lack of time these measurements were not done. Further measurements are to be done shortly. Some of this information can be derived from the frequency response curves which were shown.

A quick examination of the 0 deg and 70 deg elevation off-axis horizontal frequency response curves indicates an approximate beamwidth (-6 dB) of 100 degs between 300 and 15 kHz at 0 degs elevation and a beamwidth of 40 degs between about 800 and 15 kHz at the 70 degs elevation angle.

4 CONCLUSIONS

The frequency response measurements of the horns performance indicate a very well behaved constant-directivity horn which gets progressively narrower as the vertical elevation angle is increased. The horn's horizontal directional pattern is quite well matched in beamwidth angles to the rectangular area as seen by the horn at each vertical elevation angle.

The defined-coverage horn can be utilized very well in situations where it can substitute for several conventional horn-driver combinations that would be required to adequately cover the rectangular region. It can only be used however where the modest acoustic output capabilities of a single horn and driver combination are adequate.

The horn only partially compensates for the inverse rolloff of sound pressure with distance in the forward-backward direction. The horn does not compensate for inverse pressure variations in the horizontal side-to-side direction. In this sense the horizontal pattern provided by the horn is very similar to the patterns provided by more conventional constant-directivity horns.

5 ACKNOWLEDGEMENT

The idea for this horn resulted from a conversation with Ronald Means (VP Professional Division of JBL). Thanks go to Frank Garcia for construction of the prototype horn, Mike Park for the acoustical measurements, and to Karen Kritzberg for typing the manuscript.
6 REFERENCES

Fig. 1. Depiction of an acoustic source providing coverage for a rectangular area from an oblique angle. A rectangular coordinate \((x,y,z)\) system is shown with the \(x\) axis bisecting the rectangular \(L\) by \(W\) unit area which is on the \(x-y\) plane. The plane is shifted in the positive \(x\) direction by \(L_1\) units. The source is \(H\) units above the \(x-y\) plane on the \(z\) axis over the center line of the rectangle. Definitions for elevation angle alpha \((0^\circ\) straight down\) and horizontal coverage angle beta are shown.
Fig. 2. Illustration of the chosen system parameters for the horn designed in this paper. The horn is one unit above and centered over the narrow end of a 2 by 2.75 unit flat rectangular region. A 90 deg horizontal coverage is required to cover the end of the rectangle immediately below the horn. A much narrower coverage of roughly 38 degs is required to cover the farthest end of the rectangle. A vertical coverage angle of 70 degs is required to cover the length of the rectangle. (a) Side view of system. (b) Perspective view of system.
Fig. 2(b) See caption on Fig. 2 (a).
Fig. 3. Engineering sketch of the defined coverage horn designed to cover the rectangular area system shown in Fig. 2. The horn provides a 70 deg. vertical coverage angle. The horizontal coverage angle changes smoothly from 90 deg at the 0 deg. elevation (vertical) angle to 38 degs at the 70 deg elevation angle in a manner that matches the sides of the rectangular area as seen by the horn at each elevation angle (see Table 1). The horn is roughly 760 mm (30 in.) long, 1025 mm (40.4 in.) high, and 780 mm (30.7 in.) wide and uses a one inch throat diameter compression driver. Operating range is roughly 500 to 16 KHz.
Fig. 4. Isometric frontal view of the defined coverage horn shown in Fig. 3.
Fig. 5. Photos of the experimental prototype defined-coverage horn whose drawing is shown in Fig. 3. The constructed model includes substantial wooden rear structure which is not shown in Fig. 3. Also shown is the wooden rotation fixturing. (a) Front view of horn with the narrow-angle long-throw end pointing down. (b) Front oblique view with the wide-angle, short-throw end pointing down. (c) Side view of the horn in normal operating position with $0^\circ$ elevation angle pointing straight down. (d) Rear oblique view with narrow-angle end down showing compression driver.
Fig. 5 (c), (d). See caption on Fig. 5 (a).
Fig. 6. Frequency response (at 0 deg elevation and 0 deg horizontal angle) and impedance curve of the horn shown in Fig. 3 driven by a JBL Model 2425 compression driver. The measurement was done over the ground plane at a distance of 2 meters with 2.83 VRms applied (1 watt 8 ohms). This corresponds to a 1 watt/1 meter curve taken in free space. Due to the constant directivity nature of the horn, the response curve follows the power response of the driver.
Fig. 7. Family of horizontal off-axis frequency response curves of the horn shown in Fig. 3 taken at several selected vertical elevation angles between 0 and 70 degs. A set of 8 off-axis horizontal curves were taken at all angles between 0 and 70 degs in steps of 10 degs at each elevation angle. Same test conditions as Fig. 6.

Horizontal off-axis curves are shown for elevation angles of (a) 0 degs, (b) 10 degs, (c) 20 degs, (d) 30 degs, (e) 40 degs, (f) 50 degs, (g) 60 degs, and (h) 70 degs. Note the gradual narrowing of the horizontal coverage as the elevation angle increases.
Fig. 7 (c), (d). See caption on Fig. 7 (a).
Fig. 7 (e), (f). See caption on Fig. 7 (a).
Fig. 7 (g), (h). See caption in Fig. 7 (a).
Fig. 8. Composite set of frequency response curves of the horn shown in Fig. 3 taken at 0 deg horizontal and 8 elevation angles between 0 and 70 degs in steps of 10 degs. Same test conditions as Fig. 6. The lowest curve is the 0 deg elevation response while the highest curves are the 40, 50, and 60 deg responses.