The Design and Use of a Simple Pseudo Random Pink-Noise Generator*

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The current high usage of frequency-equalized sound systems in the commercial sound and recording fields places increasing demands on the user–operator of the system to ensure that the overall system holds its designed value of spectral flatness during normal day-to-day operation. An inexpensive generator of pseudo random pink noise with adequate spectral purity is described which can be used as a stable known test source for quick on-site sound-system tests. The generated pink noise is accurate enough for use with an elaborate real-time spectrum analyzer (if available) for quantitative measurements or can be used for qualitative operator subjective measurements in the absence of other test equipment.

INTRODUCTION: The advantages and disadvantages of using random noise for electroacoustic and sound-system tests has ben thoroughly documented in the literature [1]–[3]. Using noise as a test source (as opposed to a sine-wave single-frequency source) spreads the test energy evenly over the whole or selected portions of the audio spectrum to better simulate actual program material. The main disadvantage of truly random noise, as generated by gas-filled electron tubes and biased semiconductor junctions, is the inherent uncertainty and randomness associated with this type of test source. To yield measurements that are statistically absolutely accurate requires averaging over an infinite time interval. In practice, a balance has to be made between the amount of time averaging and the expected accuracy of the measured data (a high level of confidence requires a long time average and vice versa) [4].

WHITE AND PINK NOISE

The characteristics and definitions of white and pink noise will be summarized here for later comparison with the output of the described pseudo random noise generator.

White Noise

"White noise is the name given to random signals that contain constant power per *unit* bandwidth for all frequencies" (or at least for the audio frequency range). White noise can be considered a wide-sense stationary ergodic random process with absolutely no periodic components and exhibiting a flat power-density spectrum

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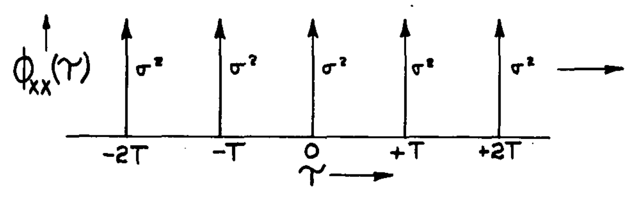

Fig. 1. Autocorrelation function of pseudo random noise of period T, assuming infinite frequency bandwidth.

[5, p. 31].[1] A flat power-density spectrum implies an impulsive autocorrelation function,

$$\phi_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,x(t+\tau)\,dt = \sigma^2\delta(\tau) \quad (1)$$

where σ^2 is the mean square value of the signal and $\delta(\tau)$ the Dirac delta function, because the power-density spectrum is the Fourier transform of the autocorrelation function [7, p. 184].

White noise as applied to audio testing usually contains the additional restraint that the amplitude probability distribution of the noise be Gaussian or near Gaussian.[2]

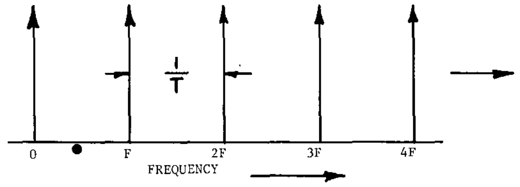

Fig. 2. Power-density spectrum of pseudo random noise.

Pink Noise

Pink noise is the name given to random signals that contain constant power per *percentage* bandwidth for all frequencies. Pink noise is usually obtained by passing white noise through a filter that has a transfer characteristic as follows:

$$H(s) = \frac{E_{\text{out}}(s)}{E_{\text{in}}(s)} = \frac{K}{\sqrt{s}} \quad (2)$$

or

$$H(j\omega) = \frac{K}{\sqrt{j\omega}} = \frac{K}{\sqrt{\omega}}\,e^{-j(\pi/4)} \quad (3)$$

where K is an arbitrary constant.

This transfer function has a decreasing amplitude characteristic of -3 dB per octave and a constant lagging phase shift of $\pi/4$ or 45°. The filter is usually synthesized with an RC network which approximates the transfer characteristic with a finite set of real poles and zeros.

Pink noise has a power-density spectrum that falls at the rate of -3 dB per octave. It is usually thought of as having a flat power-density spectrum on a constant-percentage frequency basis because spectrum analyses in audio and sound work are mostly done with analyzers with a constant-percentage bandwidth.

PSEUDO RANDOM NOISE

The problem of the very long averaging times required for accurate measurements mentioned in the Introduction can be overcome by the use of so-called pseudo random noise. Pseudo random noise has the same impulsive autocorrelation function as random white noise, but the impulses repeat with period T (Fig. 1) because the pseudo random noise signal is periodic with period T [5, p. 38]. Because of the discrete nature of the signal's autocorrelation function, the power spectrum of pseudo random noise is also discrete (Fig. 2). The power spectrum of pseudo random noise can be completely described using the Fourier series expansion of the pseudo random time waveform of period T (Fig. 3).

The periodicity of pseudo random noise implies that nearly complete information about a particular system's behavior can be obtained by averaging over only one period of the pseudo random noise.[3]

GENERATION OF PSEUDO RANDOM NOISE

The obvious way of generating a pseudo random sequence is to record a length T of purely random noise and then repeat the recording every T seconds as long as needed. The trick in this method is to capture a noise recording which has a flat enough spectrum for the period T (one could always let T get very large, but then the generated noise could hardly be called periodic because of the extremely long period).

The generation method used in the noise generator described in this paper is based on digitally generated pseudo random binary two-level white noise which is transformed into pink noise by filtering.[4] The two-level white noise is generated by applying exclusive-or feed-

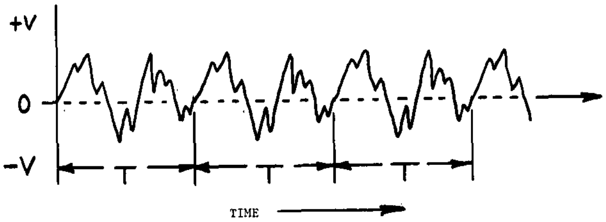

Fig. 3. Display of pseudo random noise in time domain with period T and repetition frequency $1/T$.

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[1] For a definition of these statistical terms see [6], [7].

[2] The reader will note that the original definition of white noise contained no mention of the signal amplitude probability distribution function. White noise can have 2, 3, · · ·, n levels of amplitude or can have a continuous amplitude distribution.

[3] The use of pseudo random noise for test purposes presumes that the period T of the periodic pseudo random waveform is sufficiently long to put sufficient spectral lines in the area of the spectrum under examination. For audio test purposes a period of one second, which places spectral lines every hertz, is thought to be sufficient by the author.

[4] This pseudo random generation method was first applied commercially in the two noise generators that Hewlett Packard manufactures.

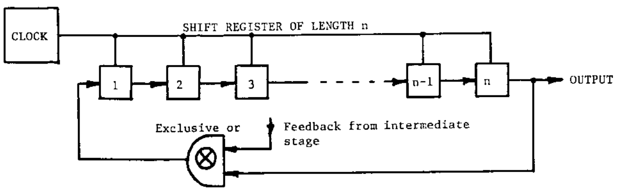

Fig. 4. Shift-register generation of recurring maximum length binary sequence of length $2^n - 1$ clock periods.

back around a shift register of length n to synthesize a recurring maximum length binary sequence of length 2^n-1 as shown in Fig. 4 [5, p. 44], [8].

The exclusive-or feedback must be taken from the last stage of the shift register and one or more of the intermediate stages to generate a maximum length binary sequence.[5]

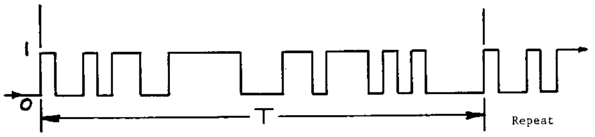

Fig. 5. Typical maximum length binary sequence (31 clock periods) for 5-bit shift register with exclusive-or feedback from the 3rd and 5th stages.

CHARACTERISTICS OF TWO-LEVEL WHITE PSEUDO RANDOM NOISE

Two-level pseudo random white noise generated by a shift register has the following characteristics (assume a shift register of length n shifting at a clock frequency F_c and generating a periodic binary maximum length sequence (2^n-1) [5, p. 43][6]:

1) flat power density spectrum to $F_c/3$ (3 dB down);

2) spectrum exhibits sin X/X behavior with zeros at F_c, $2F_c$, $3F_c$, etc.;

3) periodic with period $T = (2^n-1)/F_c$ seconds;

4) spectral lines separated by $F = 1/T = F_c/(2^n-1)$ Hz.

A time sequence and power-density spectrum for typical pseudo random two-level white noise is shown in Figs. 5 and 6, respectively. Because of the two-level nature of this type of pseudo random noise, the amplitude distribution for this waveform exhibits two impulses at the upper and lower levels of the waveform (Fig. 7).

PINK NOISE FROM TWO-LEVEL WHITE NOISE

Two-level white noise does not simulate naturally occurring noise because the amplitude distribution is not Gaussian (see Fig. 7). To generate pseudo random white noise with a nearly Gaussian probability distribution function from two-leved pseudo random white noise requires low-pass filtering with a filter cutoff frequency of about $F_c/20$ [8]. This resultant low-pass filtered white noise could in turn be sent through a pink noise filter [Eqs. (2), (3)] to generate very accurate pseudo random pink noise with all the proper characteristics.

For purposes of this specific noise generator design it was decided that the above outlined method would be too complex for a simple generator. The author determined by experiment that sufficiently accurate pink noise could be generated from two-level white noise by sending the two-level noise directly to the pink noise filter, using a clock frequency of about 64 kHz [four times the required upper cutoff frequency (16 kHz) of pink noise] and a shift register of length $n = 16$.

ACTUAL DESIGN

Block Diagram

Block diagram of noise generator is shown in Fig. 8.

Block Synthesis

Clock

A form of complementary symmetry transistor blocking oscillator was used here because of the ease of frequency adjustment and the fast output pulse. The schematic of the basic oscillator is shown in Fig. 9.

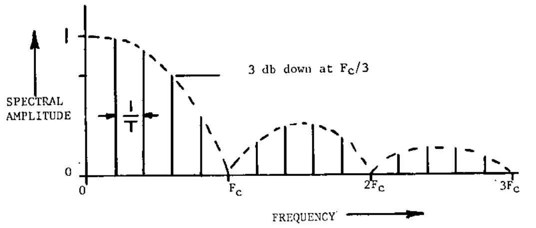

Fig. 6. Power spectrum of binary maximum length sequence repeated at period $T=(2^n-1)/F_c$, where F_c is the shift-register clock frequency.

Shift Register

The shift register was implemented with 8 dual D type flip-flops. For a 16-bit shift-register noise generator, feedback must be taken from the 3rd and 16th stages. This requirement excludes the designer from using an all in one 16-bit shift-register integrated circuit, unless the output of each individual flip-flop is available.

Exclusive-Or Gate

The exclusive-or gating function is shown in the following truth table.

In 1	In 2	Output
0	0	0
0	1	1
1	0	1
1	1	0

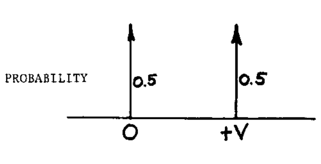

Fig. 7. Amplitude probability distribution of two-level pseudo random noise with low-level zero volts and high-level V volts.

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[5] Davies [5, p. 86] gives a tabulation of the required feedback points for register lengths up to $n = 10$.

[6] Further information on the properties of binary maximum length sequences may be found in the Appendix.

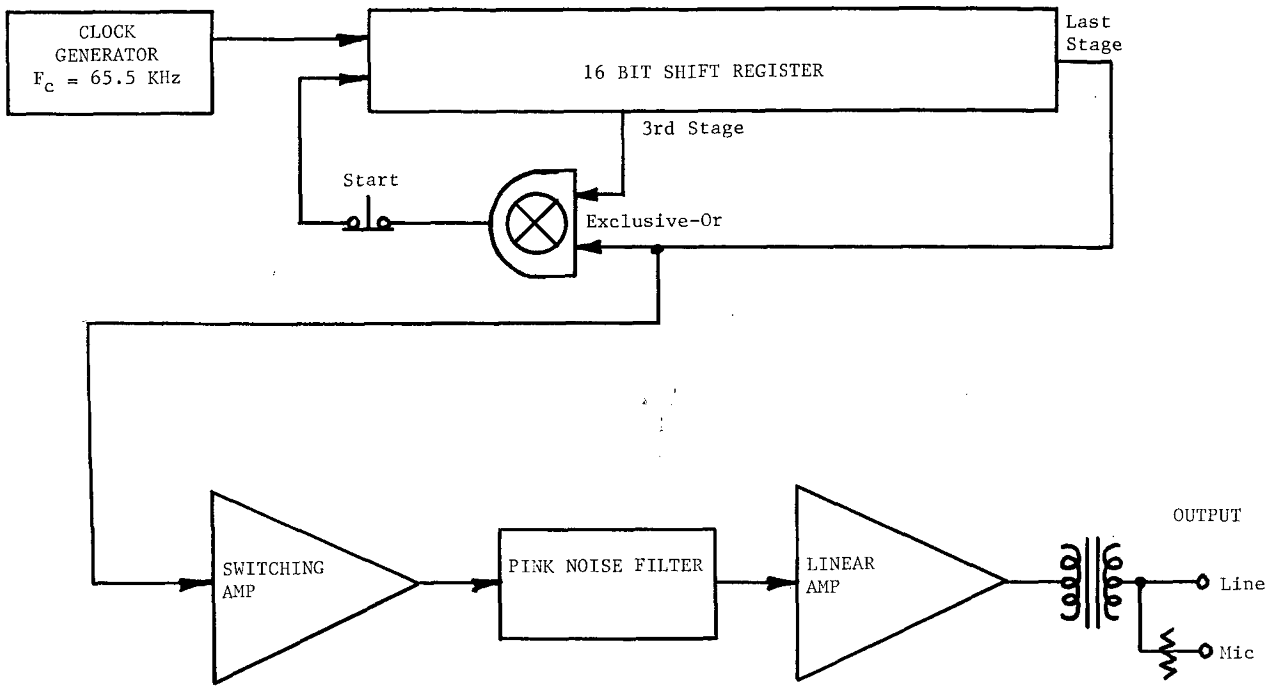

Fig. 8. Block diagram of pseudo random pink noise generator.

The table shows that the output is high when the inputs are dissimilar. This exclusive-or function was synthesized with two gates from a quad exclusive-nor gate integrated circuit the author had on hand.

Switching Amplifier

To maximize the output of the pink noise filter, the level of the binary output from the shift register was raised from 5 to 18 volts peak to peak. A conventional single transistor inverter was used for this purpose. Providing amplification at this point in the chain is efficient because it is much easier to design and implement a two-level switching amplifier than a linear amplifier.

Pink Noise Filter

The pink noise filter response (2), (3) was approximated by a passive *RC* low-pass filter with zeros distributed on the real axis. The actual filter used is an approximation with 5% tolerance components of General Radio's pink noise filter 1390-P2. The author did not design this noise filter. The noise filter is shown in Fig. 10.

Linear Amplifier

The level from the output of the pink noise filter was increased to line level by a single-transistor class *A* amplifier with an added emitter follower impedance shifter. The output was then passed through a transformer so that a balanced output would be available.

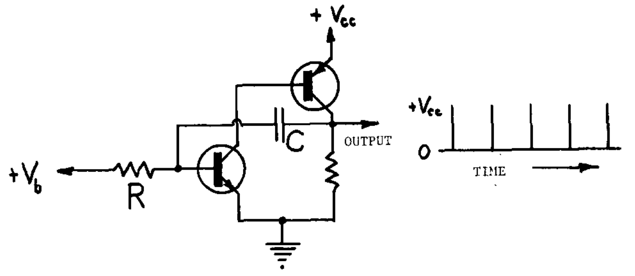

Fig. 9. Complementary symmetry transistor blocking oscillator used as clock generator for shift register.

ANALYSIS OF GENERATOR OUTPUT

One-Third-Octave Spectrum

Fig. 11 shows the one-third-octave frequency response spectrum of the generator output as measured with a Bruel and Kjaer model 2112 spectrometer and model 2305 graphic level recorder. Except for a reduced output at the one-third octave centered at 63 Hz of −3½ dB, the generator output exhibits a response of ±1 dB from 25 Hz to 25 kHz with usable output to 40 kHz. The response anomalies in the low-frequency range are mainly a result of using nonprecision components in the pink noise filter and a relatively low-frequency clock in the shift register generator. Also evident in the low-frequency spectrum are the one-second pulsations which show on the graph as ripple in each individual one-third-octave band.[7] Fig. 12 is an identical

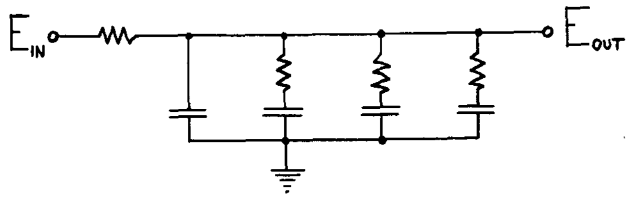

Fig. 10. General Radio model 1390-P2 *RC* pink noise filter used to filter two-level white noise into pink noise.

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[7] The paper speed for these spectrum measurements was 1 mm/s, which causes the spectrometer to dwell for 5 seconds at each one-third-octave band.

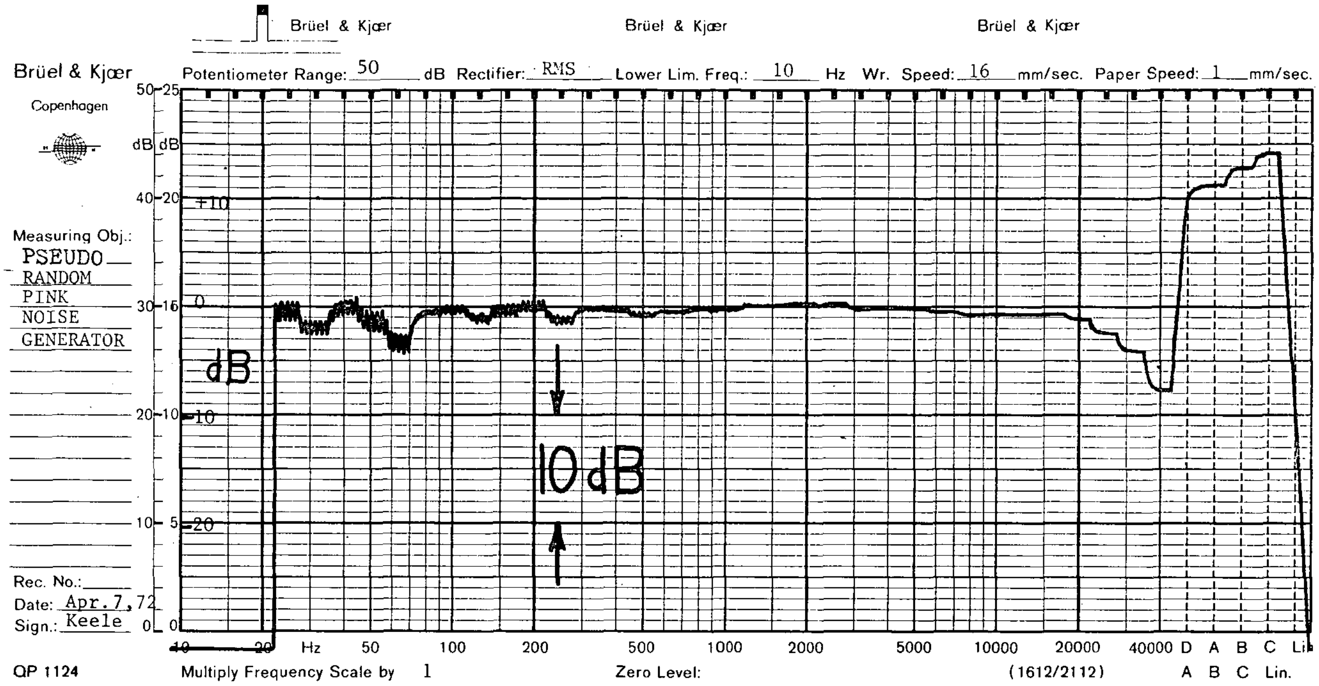


Fig. 11. One-third-octave frequency response of output of pseudo random pink noise generator. The spectrum starts at the band at 25 Hz and ends at 40 kHz. The hump at the end of the spectrum is a measurement of the output for the sound-level-meter weighting filters *A*, *B*, *C*, and linear. Writing speed 16 mm/s; the spectrometer dwells for 5 seconds at each one third octave.

spectrum measurement on a truly random noise generator which is included in this paper for comparison.

Short-Term Averaged Time Outputs

The generator's short-term rms averaged time output for several selected one-third-octave bands is illustrated in Fig. 13. The graphs cover a ten-second interval of generator output to show the repetitive nature of the generated noise. The effective averaging time for these sequences is 1.80 seconds (paper speed 10 mm/s, writing speed 63 mm/s). The reader must note that these graphed time sequences are not in time synchronization, which means that no inferences can be made about the relative peak occurrences between different one-third-octave bands. Visual observation of the generator output on a real-time one-third-octave analyzer indicates that the fluctuations of the short-term averaged output of all the one-third-octave bands above 200 Hz are essentially uncorrelated, but that below this frequency the fluctua-

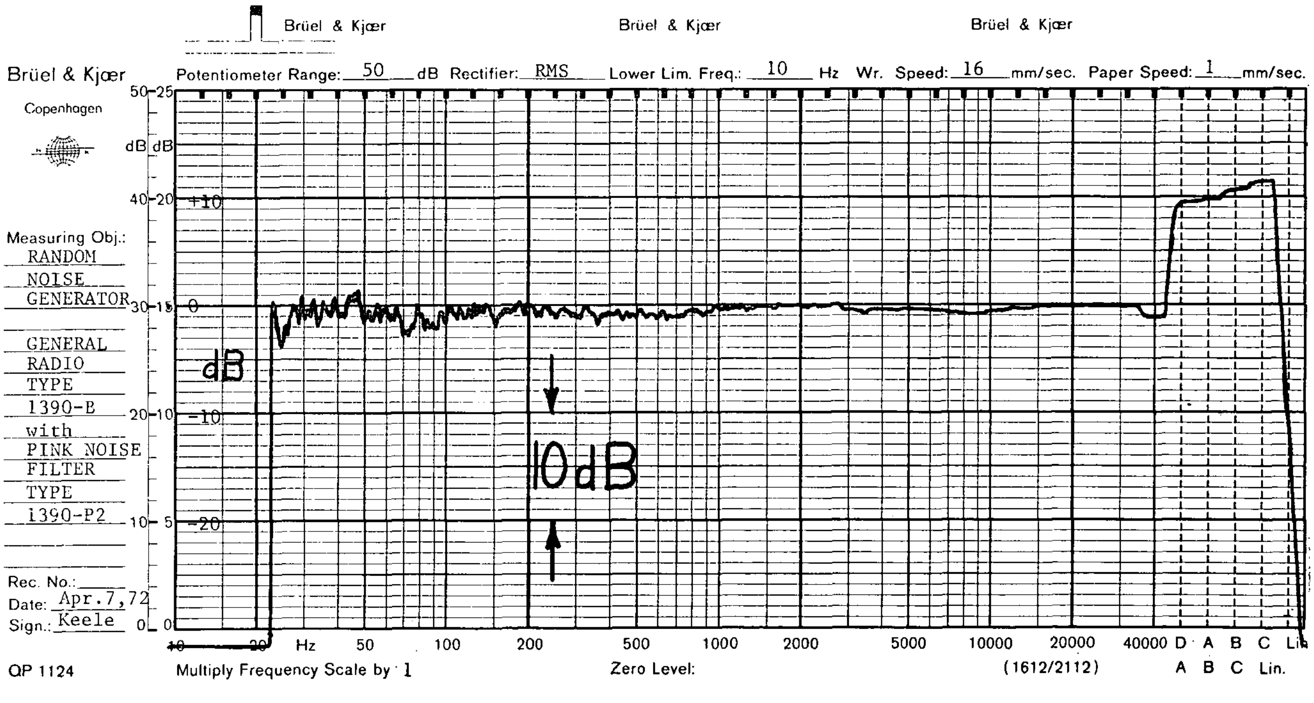
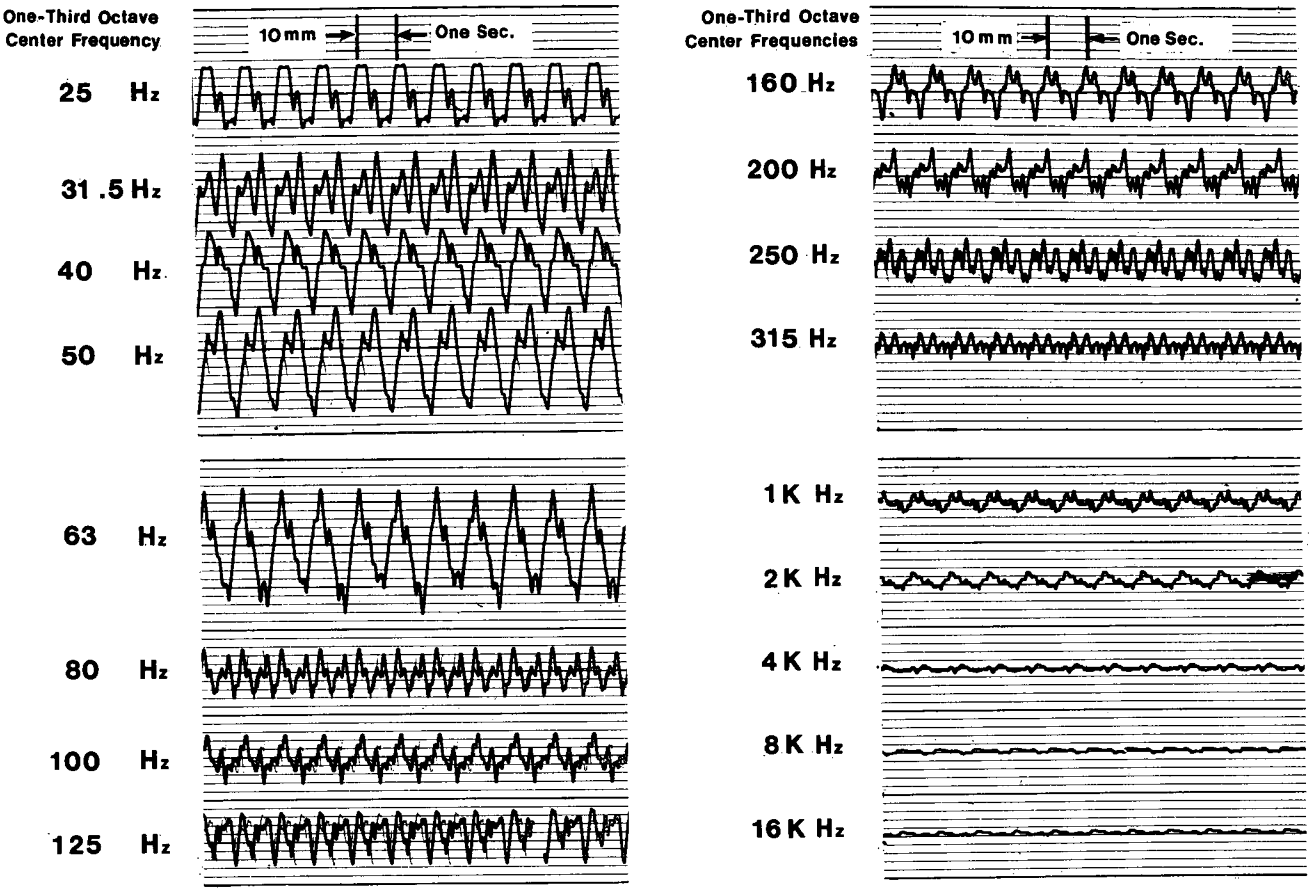


Fig. 12. One-third-octave frequency response of output of a truly random noise generator (see Fig. 11 for applicable notes).



a. b.

Fig. 13. Pseudo random noise generator short-term averaged rms time output for selected one-third-octave bands centered at the indicated frequencies. Vertical scale 1 dB/minor div, writing speed 63 mm/s, paper speed 10 mm/s.

tions are somewhat time synchronized at a 1-Hz rate. This 1-Hz low-frequency variation makes the audible pink noise sound as though a very large heart were beating behind a backdrop of radio static.

The generator short-term averaged output on an all-pass wideband basis is shown in Fig. 14 along with an identical measurement on a true random pink noise generator. These measurements indicate the time variation one would expect if a standard VU meter was used to monitor the outputs of the respective generators (4.5-dB variation for pseudo random and 3 dB variation for true random).

Instantaneous Time Output

The generator's output as viewed on an oscilloscope is very similar to true random noise except for the one-second repetitiveness. A single short positive and short negative peak of +2.4 volts and −3.7 volts, respectively, occurs once each second. The measured average voltage is 0.49 volt ac while the rms voltage is 0.62 volt ac. The positive and negative peak-to-rms voltage ratios are 3.87 and 5.97, respectively.

Amplitude Probability Distribution

The pseudo random pink noise generator amplitude probability distribution is shown in Fig. 15. Fig. 16 shows an identical measurement on a precision random noise generator. These distribution measurements were taken on a Digital Equipment Corporation digital computer model PDP 15/40. The curves are an average of 256 000 samples, taken at a rate of 25K samples per second, distributed into 200 memory registers of 50-mV size over a range of ±5 volt.

The true random pink noise generator distribution curve (Fig. 16) exhibits a nearly perfect Gaussian curve with excellent symmetry and zero mean.[8] The pseudo random generator curve (Fig. 15) exhibits a skewed near Gaussian curve with fair symmetry (positive and negative excursions are not equal) and a +65-mV mean. The rms voltage level (standard deviation) was approximately 0.6 volt for both distribution curves.

USE AND APPLICATION

There are many uses for an inexpensive generator of pink noise in the audio and sound-system field. Currently the author has the generator built into a portable real-time spectrum analyzer system for use by the uni-

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[8] A Gaussian probability distribution for the pink noise is expected because if any Gaussian process or waveform is operated on by a linear system, the output of the system will also be Gaussian [7].

versity's audio operations personnel in checkout and equalization of sound systems. The reader is referred to several references on the use of random noise for test purposes [1], [2], [5]. The author will just outline a few of the possible uses for the type of generator outlined in this paper.

In the Recording Studio

Before the start of a recording session a short burst of the generator output could be recorded on each track of the multitrack mastering recorder master tape to identify the frequency response of the tape and channel at the time of the original recording. Assuming this tape were to be pulled from the archives five years later for a mixdown, the track frequency response would be accurately identified (assuming a spectrometer of some kind is available), no matter what the condition of the record or playback machine or what kind of degradation the tape itself had undergone because of environmental storage conditions. Once the response of the tape had been identified, the proper equalization could be applied to compensate for the errors. This same kind of technique could be applied to the whole mixdown duplicating process so that when the consumer received his tape or record he could precisely identify the playback frequency response and hence the required equalization needed to compensate the response.

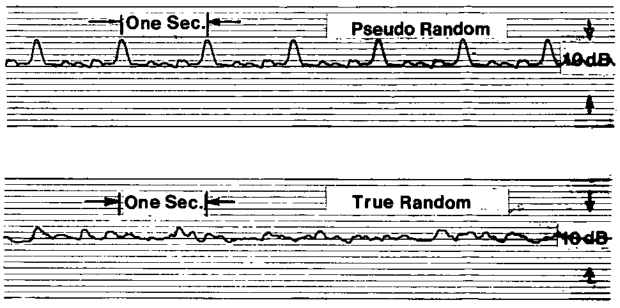

Fig. 14. Comparison of all-pass (10 Hz to 100 kHz) short-term averaged rms outputs for pseudo random (author) and true random (General Radio 1390-B with 1390-P2 pink noise filter) pink noise generators. Writing speed 100 mm/s, paper speed 30 mm/s, lower limiting frequency 10 Hz.

Audio Systems in General

In the absence of sophisticated analysis equipment, such as a real-time spectrum analyzer, the user–operator of a complex audio system can derive usable subjective information from having a pink noise source available. Because of the equal energy per octave, extremely wide frequency range, and short-term spectrum stability characteristic of pseudo random pink noise, the generator output is very useful as a subjective program test source. Pink noise is the best type of test program material

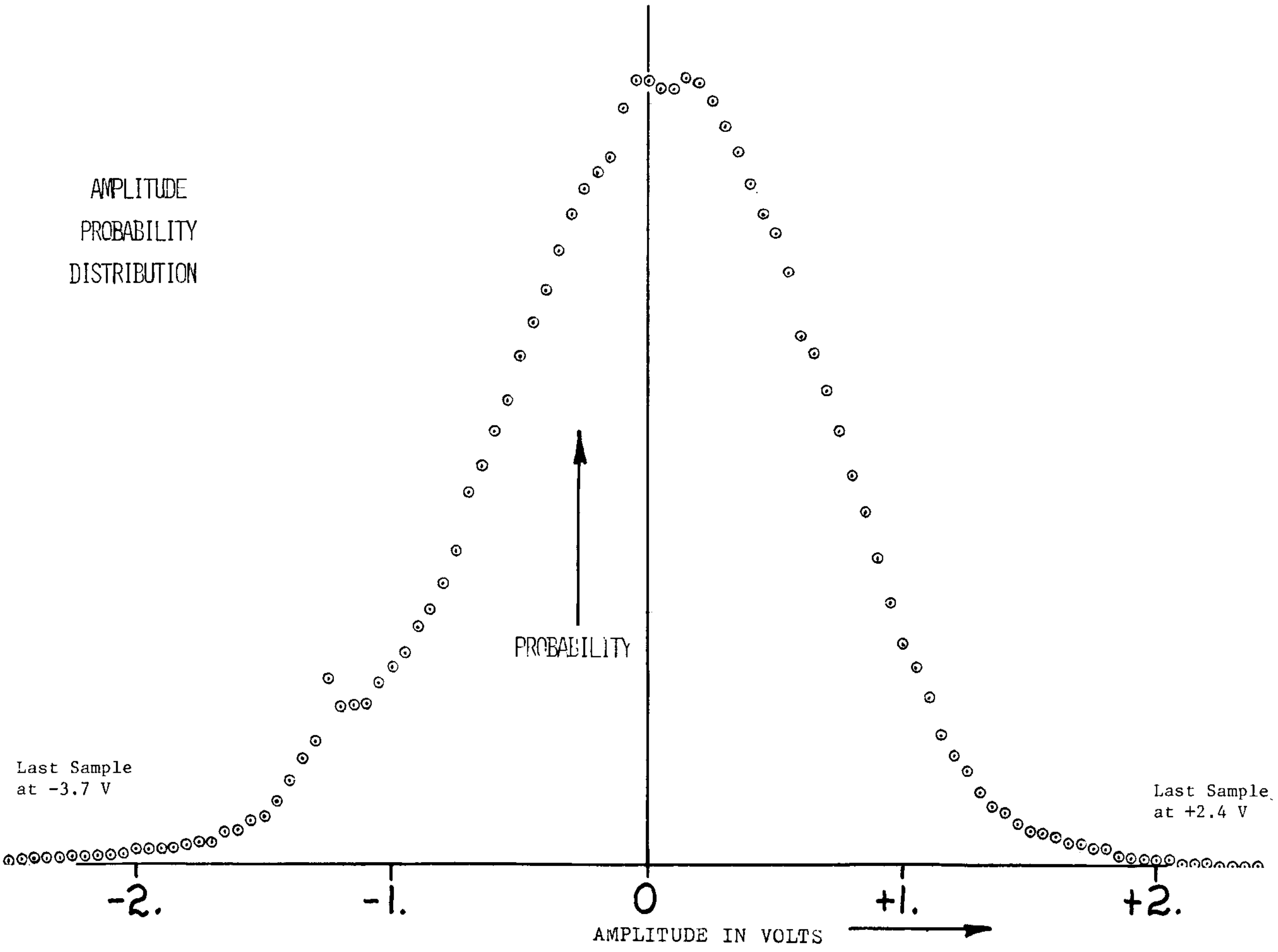

Fig. 15. Amplitude distribution of output of pseudo random pink noise generator as measured on a PDP 15/40 computer. RMS value or standard deviation of waveform 0.61 volt.

when any kind of aural frequency response evaluations are to be made such as tests of equalizers, channel comparisons on multichannel consoles, and comparisons of speaker systems.

Absolute subjective frequency response determinations can be made by listening to the pink noise through a good pair of electrostatic headphones. Aural headphone comparison of the direct output of the generator (through a good flat amplifier) and the output as modified through an audio link will reveal much about the frequency response and overload clipping characteristics of the link.

CONCLUSIONS

This paper has described a comparatively simple digital pseudo random audio pink noise source which is suitable for all kinds of audio system measurements. Using only a 16-bit shift register with associated gates, filters, and amplifiers, it generates an equal energy per octave signal with adequate spectral frequency response and amplitude distribution. Because of the digital noise generation method used, the output time sequence repeats on a one-second pseudo random basis which assures the person making audio measurements that nearly complete information about the system under test may be gathered in 2 to 3 seconds of measurement time.

APPENDIX

Some properties of binary maximum length sequences [5, p. 59] are as follows:

1) An n-bit shift register generator generates a sequence, N digits long, where $N = (2^n - 1)$.

2) The N-digit sequence contains $(N+1)/2$ logic 1 states and $(N-1)/2$ logic 0 states, which means that the sequence contains one more logic 1 states than logic 0 states.

3) A given n-stage shift register generator generates all the possible n-tuple binary numbers once and only once except for the all-zero n-tuple which does not appear at all.

4) "If successive occurrences of one of the states of a binary maximum length sequence are called 'runs,' and these runs are tabulated, it may be seen that there are $2^{n-1} = 2^{(n-1)}$ or $(N+1)/2$ runs in sequence, of which one half are one digit long, one quarter are two digits long, one eights are three digits long, etc., provided that the number of runs of a given length so indicated is greater than one. There are an equal number of runs of either state, except that there is a run of n logic 1 states but no run of logic 0 states. There is, however, a run of $(n-1)$ logic 0 states, but no run of (n-1) logic 1 states."

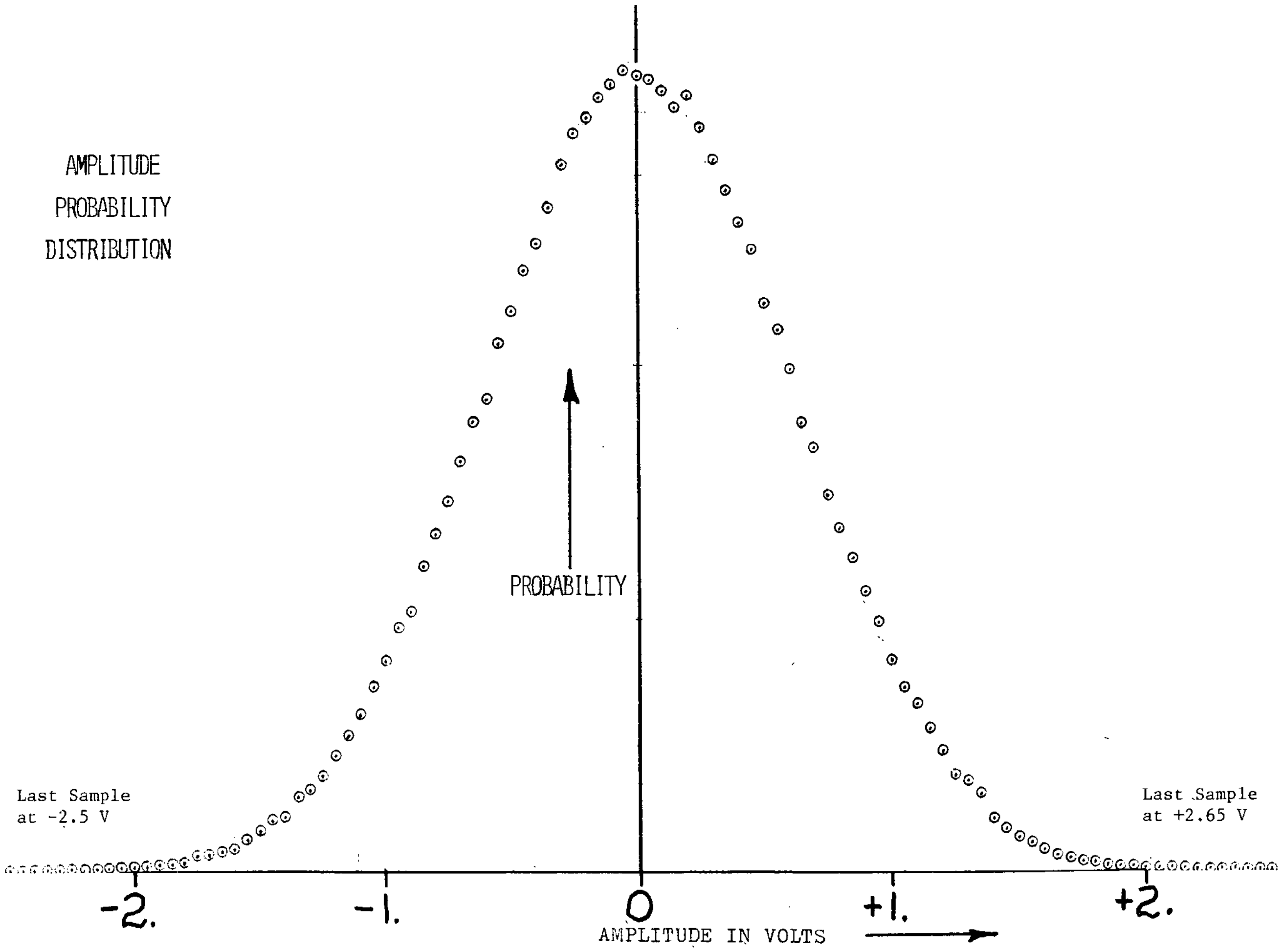

Fig. 16. Amplitude distribution of precision random pink noise generator as measured on a PDP 15/40 computer. This HP 8057A generator outputs a 2-second pseudo random sequence of nearly perfect Gaussian characteristics.

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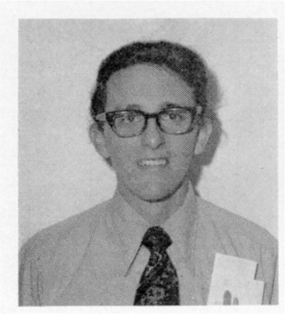
THE AUTHOR



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In June of 1972, Mr. Keele joined Electro-Voice, Inc., where he works as a loudspeaker development engineer and is continuing work on his masters degree with company sponsored research in the area of loudspeaker horn design. Mr. Keele is a member of the Audio Engineering Society.