Application of Linear-Phase Digital Crossover Filters to Pair-Wise Symmetric Multi-Way Loudspeakers Part 1: Control of Off-Axis Frequency Response

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ABSTRACT

Various methods exist for crossing over multi-way loudspeaker systems. These methods include those loosely classified as Linkwitz-Riley filters, constant-voltage filters, and D'Appolito configurations. All these methods do not provide broad-band constant-beamwidth or constant-directivity operation because their vertical radiation patterns change shape as a function of frequency. This paper describes a simple, non-iterative linear-phase crossover filter design technique that provides uniform frequency responses vertically off-axis for a given multi-way loudspeaker. Distances between the individual drivers, and desired off-axis attenuation are prescribed as input parameters for the design process, the outcome of which is a set of crossover frequencies and unique filter frequency responses in each band. In order to obtain wide-band constant-beamwidth, a loudspeaker array configuration composed of a single central tweeter surrounded symmetrically by pairs of lower-operating-frequency transducers arranged in a vertical line is required. Practical implementation issues are outlined in the paper by means of various design examples. Two design methods are presented in in two-parts: Part 1: a general method which emphasizes flatness of arbitrary off-axis frequency responses and Part 2: a simplified method that emphasizes frequency uniformity of beam shape and coverage angle (vertical beamwidth) of the polar patterns.

0 INTRODUCTION

Uniform and smooth off-axis responses are widely accepted as key features of a successful loudspeaker design [1]. Ideally, one wishes to achieve flat, frequency-independent amplitude responses at any measured point in space.

A common design practice is to accept non-ideal behavior, such as interference due to path-length differences in multi-way speakers, as unavoidable, and then try to optimize the apparent sound quality by modifying crossover parameters – a process commonly known as *voicing*.

There are efforts to circumvent the problem and approach the ideal more closely. Keele [2] has presented a novel array design that features excellent control over the radiated sound field. However, a high number of high-quality wide-band drive units are required, and a long curved array is not always suitable for domestic use. In [3] two different approaches are shown – a distributed mode loudspeaker, and a two-way system with large high-frequency waveguide and digital, brick-wall crossover. Drawbacks are high distortion with the former, and remaining crossover artifacts such as pre-ringing with the latter. Recently, Shaiek *et al* [4] have presented a *high-end* four-way full-bandwidth coaxial source. Despite of the very high complexity and cost of their design, the achieved off-axis responses appear very irregular and require iterative, sub-optimal equalization methods.

Van der Wal's article on logarithmic arrays [5] describes a design algorithm related to our method. Here, optimum zero-phase low pass filters provide frequency-dependent array aperture, in order to achieve constant-directivity. However, it is not a multi-way crossover design, since all drivers are required to reproduce the low-frequency band.

In our new technique described here[1], a DSP-based crossover is designed to work with a loudspeaker array composed of a single central tweeter surrounded symmetrically by pairs of lower-operating-frequency transducers arranged in a vertical line. Each pair of drivers at the same distance from the center is driven by a separate crossover channel, including the single central tweeter. At any specific frequency, only one pair or at most two pairs of speakers are operating simultaneously (the single central tweeter is the sole exception which operates by itself at high frequencies). This feature of the new technique allows the design method to apply both to equally- and unequally-spaced pairs of drivers.

In Part 1, the design procedure for the new technique is based on specifying a crossover frequency-response shape that forces a flat frequency response at a specified vertical off-axis angle. When thus specified, frequency responses at other vertical off-axis angles are found to be reasonable flat as well. In Part 2 of this paper we

[1] Patent applied for May 6, 2005 (US 20060251272).

present a somewhat-simplified alternate design procedure that emphasizes frequency uniformity of beam shape and coverage angle (vertical beamwidth) of the polar patterns. This paper illustrates the two different design procedures by applying the former design technique to the design of multi-way loudspeaker monitors in Part 1, and the latter to the design of broad-band constant-beamwidth vertical line arrays in Part 2.

In Part 1, we discuss the performance of some commonly used crossover alignments in section 1, then introduce our technique in section 2, briefly cover filter implementation issues in 3, and close with a summary in and section 4. In Part 2, we develop an alternate design method for constant-beamwidth line arrays.

1 TRADITIONAL CROSSOVER ALIGNMENTS

We will employ a three-way loudspeaker design as depicted on the left in Fig. 1 throughout this section to illustrate the frequency responses of conventional crossovers. A small neodymium tweeter with low resonance frequency is used to minimize the distance to the midrange and allow a low crossover point. For a given crossover filter, we compute vertical off-axis responses by applying circular piston models for the transducers, and compute the complex sum of the respective terms after multiplication with the crossover transfer functions. In the asymmetric case, different sound pressure levels result for angles above and below the main axis. We also consider a symmetric arrangement of transducer pairs around the tweeter as illustrated on the right in Fig. 1, as proposed by d'Appolito [6]. We restrict our considerations to the far field, where the observation distance is large compared with the dimensions of the loudspeaker.

The sound pressure of a single monopole x_i at positive angles (upwards) is

$$C_{u,i} = \frac{1}{2}\exp(-j\,2\pi\, d_i / \lambda), \quad (1)$$

and negative angles (downwards)

$$C_{d,i} = \frac{1}{2}\exp(+j\,2\pi\, d_i / \lambda). \quad (2)$$

The sound pressure of a source pair is the sum of both:

$$C_i = C_{u,i} + C_{d,i} = \cos(2\pi\, d_i / \lambda) \quad (3)$$

where (see Fig. 2)

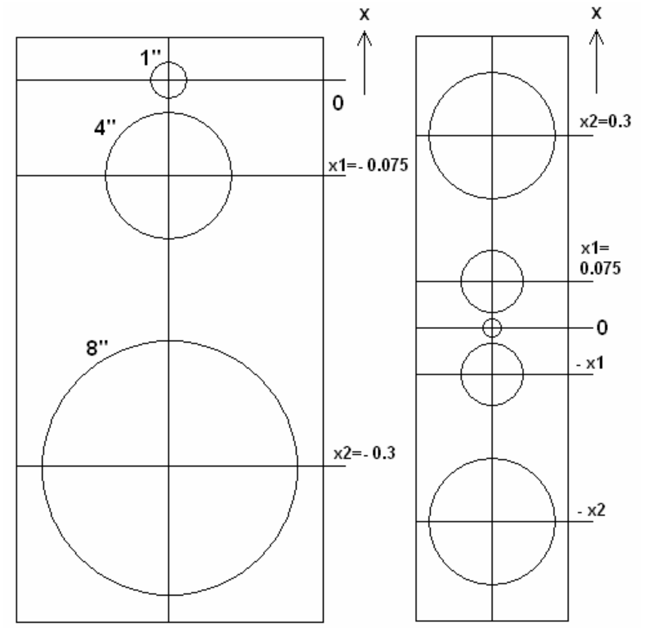

Figure 1: Speaker system configurations analyzed in this paper: conventional (left) and pair-wise symmetric three-way layout (right). X-axis (vertical) distances in meter.

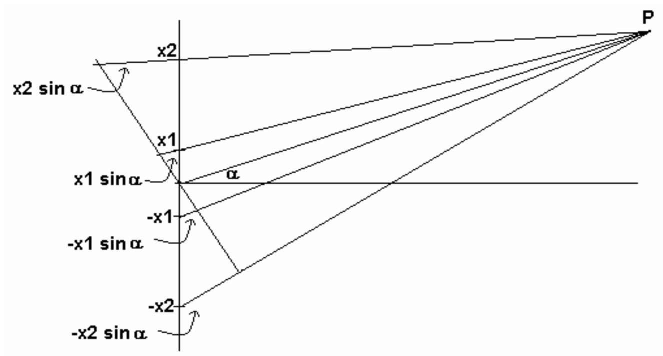

Figure 2: Source locations and path differences for two pairs of point sources located symmetrically about a center position at vertical positions ±x1 and ± x2.

$$d_i = x_i \cdot \sin\alpha, \quad \lambda = c/f, \quad c = 346\, m/\sec \\ i = 0,1,2; \quad x_0 = 0. \quad (4)$$

The far field sound pressure level of the three-way loudspeaker at angle α is

$$H_\alpha = D_{0,\alpha} H_{HP} + D_{1,\alpha} H_{BP} C_1 + D_{2,\alpha} H_{LP} C_2, \quad (5)$$

where $D_{i,\alpha}$ is the attenuation of the i-th transducer at angle α, using the first order Bessel function J_1

$$D_{i,\alpha} = 2\frac{J_1(2\pi \cdot d_i \cdot \sin\alpha \cdot f / c)}{2\pi \cdot d_i \cdot \sin\alpha \cdot f / c}, \quad (6)$$

C_i, $i=0,1,2$, the monopole pressure response according to (1), (2) or (3), and H_{HP}, H_{BP}, H_{LP} are the transfer functions of the high-pass, band-pass and low-pass, filters respectively.

Example three-way crossover filter frequency responses for several crossover types are shown in Fig. 3. The chosen crossover frequencies are *250Hz* and *1500Hz*.

Three well-known alignments are illustrated here: a) 4th-order Linkwitz/Riley, b) 2nd-order constant-voltage, and c) 8th-order notched. For comparison, our method is shown in d) and is described later in section 2. All frequency response functions have been equalized to flat on axis.

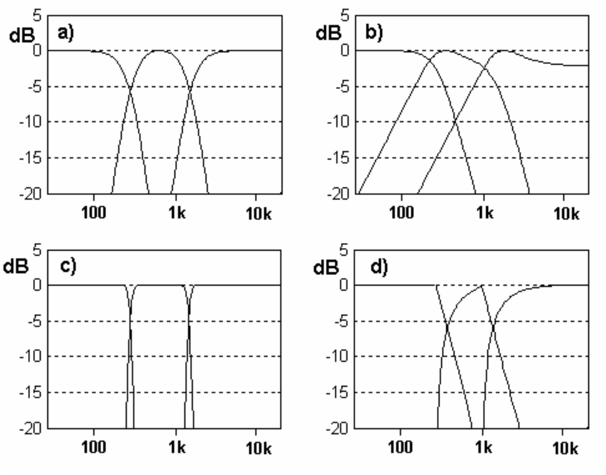

Figure 3: Frequency responses of several three-way crossover filters. a) 4th-order Linkwitz/Riley; b) 2nd-order constant-voltage c) 8th-order notched; and d) newly proposed linear-phase filter.

Fourth-order Linkwitz/Riley (L/R) crossovers became popular because the high- and low-pass filters are in phase, resulting in symmetric lobes without tilt [7]. Figure 4 shows on- and off-axis frequency responses for a three-way L/R design based on the two configurations shown in Fig. 1. The responses for the conventional configuration (top two graphs), show that the up-down responses *are not* symmetrical because adjacent crossovers interact with each other. Discussions of complete L/R designs, rather than just looking at isolated single crossover points, have not been widely published so far. The bottom graph illustrates the responses for the pair-wise symmetric driver layout. Although the up-down responses are guaranteed symmetrical, the off-axis responses are clearly not flat. Thus in practice, the L/R design may not provide either symmetrical up-down response or flat off-axis response, even though it is a clear improvement over a third-order Butterworth filter with phase-inverted midrange shown in Fig. 5.

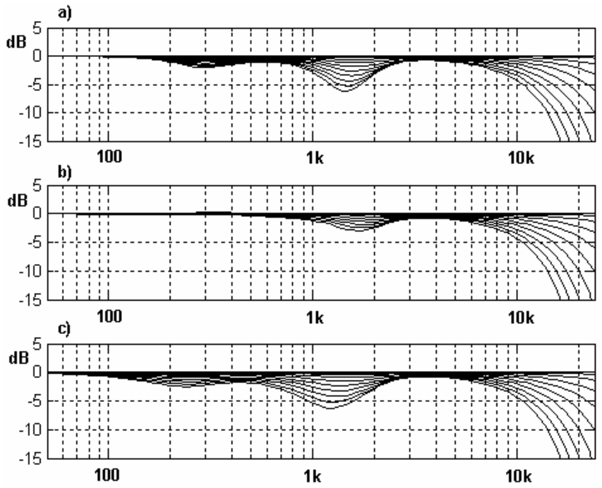

Figure 4: Fourth-order Linkwitz/Riley off-axis frequency responses for the three-way systems of Fig. 1 at 0…45° in 5° steps. a) above axis b) below axis c) pair-wise symmetric driver layout. Crossover frequencies are *250Hz* and *1500Hz*.

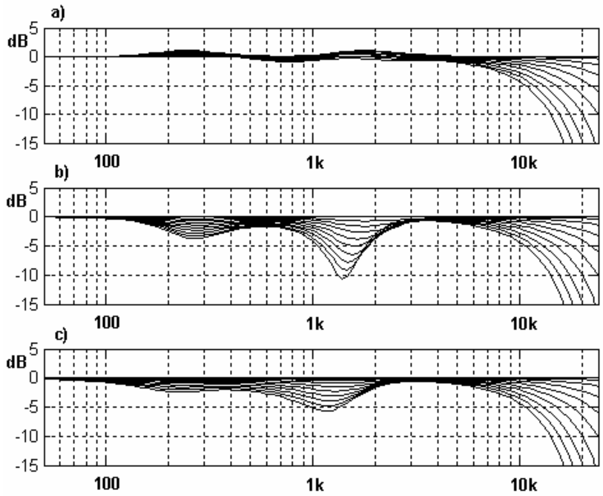

Figure 5: Third-order Butterworth inverted off-axis frequency responses at 0…45° in 5° steps. a) above axis b) below axis c) pair-wise symmetric driver layout.

An interesting result is shown in the following Fig. 6, which shows the responses for a second-order constant-voltage design [8]. This crossover provides poor results when applied to a conventional asymmetric three-way layout, but excellent results when applied to the symmetric configuration. The symmetrical response is close to an ideal point-source with little beaming. The only drawback is that transducers have to cover a wide frequency range without distortion and membrane breakup.

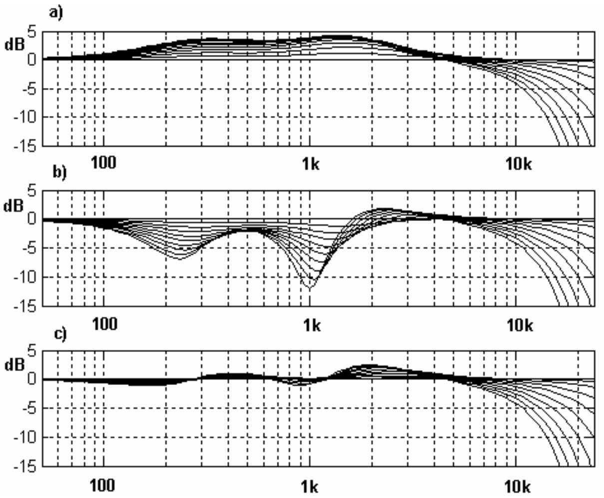

Figure 6: Second-order Butterworth constant-voltage off-axis responses at 0…45° in 5° steps. a) Above axis b) below axis c) pair-wise symmetric driver layout.

A recent article proposes to use high order "notched" crossovers [9], which are known as "Chebyshev Type II" in analog filter design literature. Figure 7, however, shows that the results are far from perfect. It seems preferable to avoid out-of-axis artifacts altogether, rather than claiming their inaudibility.

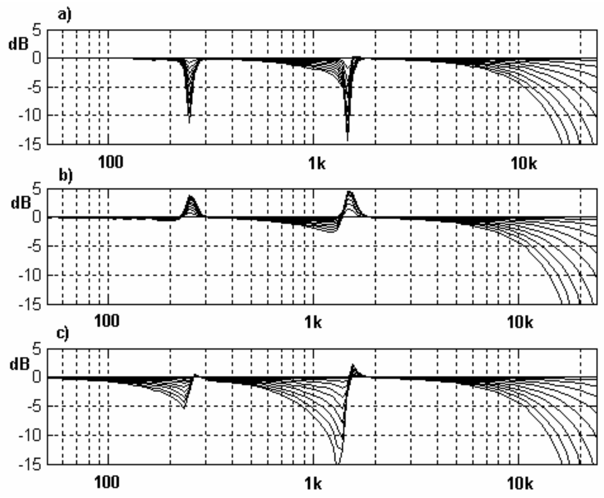

Figure 7: Eighth-order notched off-axis responses at 0…45° in 5° steps. a) above axis b) below axis c) pair-wise symmetric driver layout.

Here not further discussed is the work of Rimell and Hawksford [10], and Greenfield [11], who published new, improved digital crossover alignments. Proposed were frequency responses based on Gaussian functions, and "pseudo-analog" filters with wide transition bands and high stop-band attenuation. However, their approaches aim at minimizing crossover artifacts, rather than including off-axis path differences directly into the design. This is the basic idea of the following section.

For comparison, Fig. 8 shows the corresponding up/down frequency responses for the new linear-phase filter proposed in this paper applied to the pair-wise symmetric driver layout of Fig. 1. Except for some broadening in the 3 to 8 kHz range and narrowing at high frequencies, the response is very smooth and flat and exhibits perfect up/down symmetry.

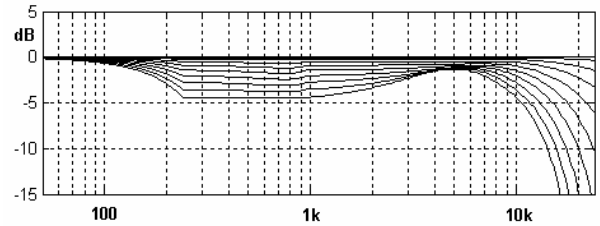

Figure 8: Off-axis frequency responses for the new linear-phase filter for three-way pair-wise symmetric driver layout at 0…45° in 5° steps (explained in section 2). In this design, the response is forced to be flat at a level of about -4.5 dB at ±45° from on axis, i.e. a = 0.6 and α = 45°.

2 CROSSOVER FILTER DESIGN TECHNIQUE

In this section, we only consider symmetric layouts of pairs of midranges and woofers arranged around a central tweeter (Figure 1, right side).

2.1 Basic Design

The far-field frequency responses C_i and C_{i+1} of two pairs of point sources i and $(i+1)$ (Eqs. 3 and 4), crossed over by yet undetermined functions $w_i(f)$ (lowpass) and $1-w_i(f)$ (highpass), respectively, is

$$H(f) = w_i(f)\cdot C_{i+1}(f) + (1-w_i(f))\cdot C_i(f),\quad i=1,2,... \tag{7}$$

We prescribe

$$H(f) = a \quad at \quad \alpha = \alpha_0 . \tag{8}$$

Parameter a is an attenuation factor that specifies the level of a specific off-axis frequency response (both up and down), i.e. the frequency response will be perfectly flat at this plus-minus off-axis angle. For example, an a of 0.316 at α = 45º, specifies that the frequency response will be flat at 45º above and below the system's axis at a level 10 dB down from on axis.

Simple algebra then yields the crossover functions

$$w_i(f) = \frac{a - C_i(f)}{C_{i+1}(f) - C_i(f)} . \tag{9}$$

The numerator is zero when

$$C_i(f) = a . \tag{10}$$

Using (3) we obtain for this case

$$f_i = \frac{c \cdot \arccos(a)}{2\pi \cdot x_i \cdot \sin\alpha_0}, \quad (11)$$

the frequencies where the lowpass crossover functions $w(f_i)$ are zero. We call f_i *"critical frequencies"*. At any angle α other than α_0, the value of the sound pressure a_α at the critical frequencies is

$$a_\alpha = \cos\left(\frac{\sin\alpha}{\sin\alpha_0}\arccos(a)\right), \quad (12)$$

combining Eqs. (7) and (11) with $w(f)=0$.

Figure 9 shows typical curves for $w(f)$ and $(1\text{-}w(f))$. They can be used in a frequency interval as crossover functions, defined by a pair of critical frequencies (here about 250Hz … 450Hz). The crossover frequency can be found where both functions cross, and takes on a value of 0.5 (-6dB) (in Fig. 9 at about 300Hz). Both crossover frequencies and interval boundaries cannot be chosen freely. They depend on the given input parameters x_i (driver locations), and the prescribed attenuation factor a at the desired angle α.

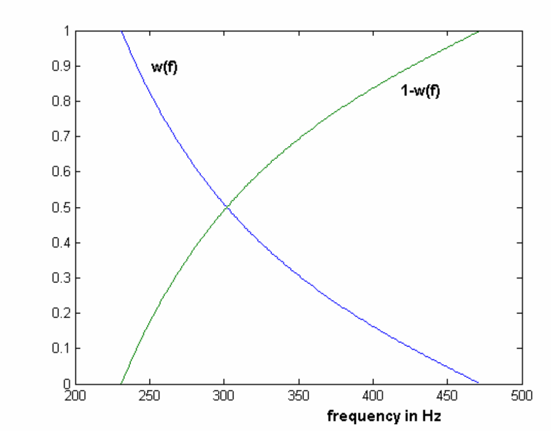

Figure 9: Crossover functions $w(f)$ and $1\text{-}w(f)$ computed from Eq. 9 that define the frequency response shape of the new crossover.

Figure 10 illustrates a six-way application. A central tweeter is located at $x_0=0$, and five pairs of midranges/ woofers at $+/-\ x_i$, $i=1…5$. We obtain a highpass, four bandpass filters characterized by four critical frequencies according to (11), and a lowpass $w_5(f)$. At a critical frequency, only one pair of transducers are active, otherwise only two pairs.

2.2 Control of Low-Frequency Response

Below the lowest critical frequency, the system response $H(f)$ equals the response of a pair of monopoles (equation 7, $w=1$, $H=C_{i+1}$), which

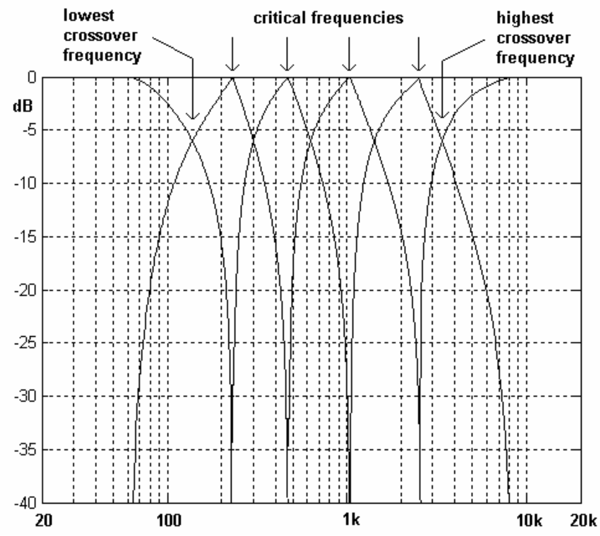

Figure 10: Crossover filters of a six-way array system designed to provide broadband flat off-axis response between roughly 200 Hz and 5 kHz.

approaches one at $f=0$. The loudspeaker becomes omnidirectional at low frequencies. In order to achieve a smoother transition to constant-directivity, we prescribe a frequency-dependent target function $a_1(f)$, to be applied to the lowest interval:

$$w(f) = \frac{a_1(f) - C_{M-2}(f)}{C_{M-1}(f) - C_{M-2}(f)}. \quad (13)$$

We can for example use a spline function, as shown in Figure 11 (M is the number of ways, here $M=6$). The corresponding critical frequency is reduced by a factor $c<1$ compared with the original one after Eq. (11). In general, Eq. (13) can be used if one wishes to approximate any non-constant directivity function.

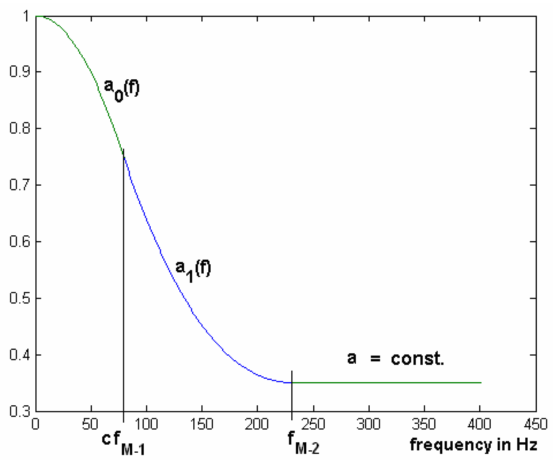

Figure 11: Low-frequency directivity target functions.

2.3 Control of High-Frequency Response

Above the highest critical frequency, the single central tweeter operates essentially on its own. In most cases, a high frequency transducer (tweeter) cannot be accurately modeled as a monopole. For instance, a waveguide might be used to extend the desired constant directivity to the upper frequency of the loudspeaker. We propose an iterative method to optimize the crossover function for the tweeter that uses modeled or measured data. This technique essentially includes the narrowing or non-uniform off-axis high-frequency response of the tweeter in the overall design.

We discretize frequency and off-axis angles of interest in the highest frequency interval:

$$f_n = f_1 + n/N(f_g - f_1), \quad n = 0...N,$$
$$\alpha_k = k/K \cdot \alpha_0, \quad k = 1...K, \tag{14}$$

then successively search real-valued *x(n)* for *n=1,2,…N* such that the error

$$e_n = \sum_k (H(f_{n-1}, \alpha_k) - a(k))^2, \tag{15}$$

with

$$H(f_n, \alpha_k) = x(n) \cdot C_1(f_n) + (1 - x(n)) \cdot H_{Tw}(f_n, \alpha_k) \quad .....(16)$$

becomes minimum. The values *a(k)* are attenuation factors at the angles α_k according to (12). H_{Tw} is the tweeter's frequency response, normalized to the response on axis.

2.4 Examples

Figures 12 and 13 present illustrative examples of the off-axis response that results from the design method.

Figure 12 shows that constant directivity has been achieved with a six-way system throughout the whole audible range, with differently prescribed beamwidth targets: a) narrow, and b) wide. Figure 12b is an example for a nearly perfect point source, with little beaming, at the expense of lower crossover frequencies.

Figure 13 shows responses up to 2kHz for the whole angular range up to 90°. Above that frequency, a tweeter with optimized directivity may take over. We confirm that the responses are forced to be flat at the prescribed angle, and are reasonably flat at other angles, with frequency-independent fixed values at the critical frequencies according to Eq. (12). The figure also shows, that by parameter choice, one can generate designs with or without side lobes, and focus on very flat responses over a limited range of angles, or reasonably flat responses over the whole range.

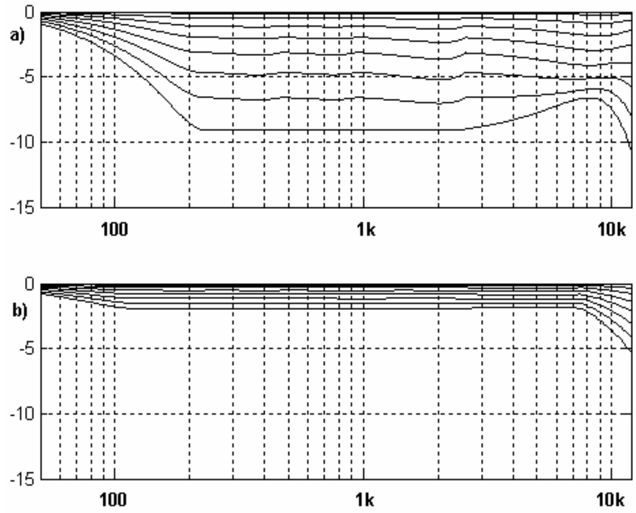

Figure 12: Frequency responses of a six-way system designed to have narrow vertical coverage (a) and wide vertical coverage (b). a) α=40, *a*=0.35 (level of -9 dB at ±40º); b) α=40, *a*=0.8 (level of -2 dB at ±40º) shown at 0…40° in steps of 5°.

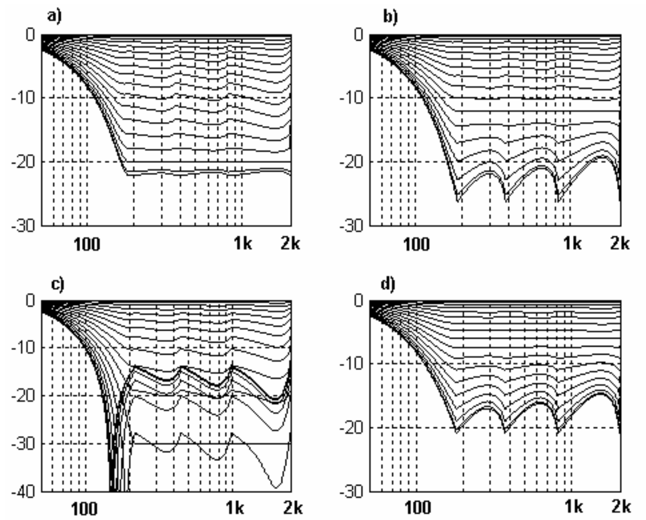

Figure 13: Graphs illustrating off-axis frequency response for four different sets of level *a* and angular α parameters over a wide angular range out to ±90°. Parameter variations include: a) α=80, *a*=0.1 (level of -20 dB at ±80º); b) α=60, *a*=0.25 (level of -12 dB at ±60º); c) α=60, *a*=0.031 (level of -30 dB at ±60º); and d) α=45, *a*=0.5 (level of -6 dB at ±45º); shown at 0…90° in steps of 5°.

The three-way example in Figure 8 has been created using x_1=0.075, x_2=0.3, a=0.6 at α=45°, and a pistonic model for the tweeter (Eq. (6) with d=0.015 m). A properly designed waveguide would help to maintain constant directivity over a wider frequency range.

3 IMPLEMENTATION AND DRIVER EQ

The zero-phase crossover functions of Eq. (7) can be approximated by linear-phase FIR (Finite Impulse Response) filters using well-known LMS or Fourier-approximation methods. In order to achieve the required linear phase response for the overall acoustic system, loudspeaker driver magnitude and phase equalisation must be incorporated in the crossover design. The simple method outlined in the following equation turned out to be effective in most cases.

$$H_{result} = H_{cross} / FFT(b_{driver}), \quad b_{result} = IFFT(H_{result}) \qquad (17)$$

Here, we divide the crossover function H_{cross} by the spectrum of the measured driver's impulse response, and obtain the final filter coefficients by applying an inverse FFT, time shifting and –gating. An example is shown in Fig. 14. Filter degrees are usually moderate, because the filters contain no passband, and wide transition bands. Multirate techniques can be employed to minimize implementation cost.

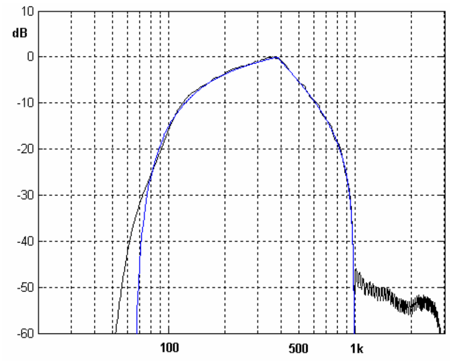

Figure 14: Approximation of a crossover filter using an FIR filter.

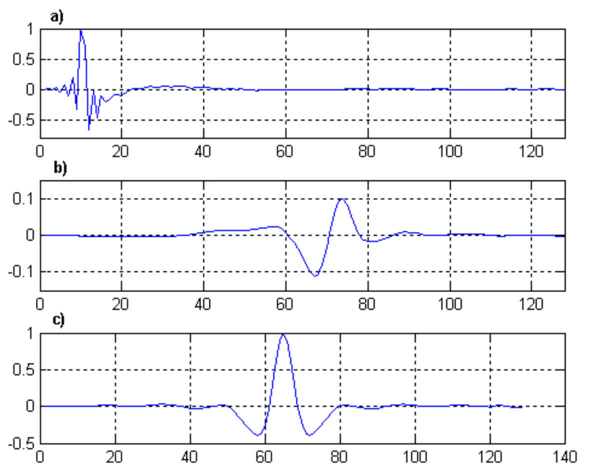

Figure 15: Impulse responses a) measured driver response b_{driver} , b) combined EQ and crossover filter impulse response b_{result} , c) acoustic impulse response.

4 SUMMARY

In this paper we described a new linear-phase DSP technique for crossing over multi-way loudspeakers utilizing pair-wise symmetric driver configurations with a central tweeter in a vertical array. The technique is based on combining the acoustic outputs of pairs of drivers to yield a flat frequency response at an arbitrary specified off-axis angle. When thus flattened, responses at other off-axis angles are found to be fairly flat as well.

In contrast to prior crossover techniques such as Linkwitz-Riley, constant-voltage, high-order notched, etc., the new technique actually maintains flat off-axis frequency response throughout most of the operating range of the speaker except at high frequencies where the single central tweeter operates on its own.

The technique produces a crossover filter frequency response with a very distinctive pointed-top shape. On either side of the point, called a critical frequency, the response rolls off rapidly and essentially shuts off at frequencies above and below the critical frequencies of the adjacent drivers. At a critical frequency, only one pair of drivers are energized. At frequencies between the critical frequencies, only two pairs of speakers are operating.

In Part 1 of this paper (this part), we described a technique that places emphasis on the flatness of off-axis frequency response. This is done by specifying an attenuation factor that specifies the level of a specific off-axis frequency response (both up and down). Thus specified, the crossover filter forces the frequency response at that specific off-axis angle to be flat and nearby off-axis angles are found to be fairly flat as well.

5 ACKNOWLEDGEMENTS

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All calculations and graphing in Part 1 were accomplished using MatLab (www.mathworks.com/) and in Part 2 using Igor Pro (www.wavemetrics.com/).

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