Audio Engineering Society
Convention Paper

Presented at the 117th Convention
2004 October 28–31 San Francisco, California



# Maximum Efficiency of Compression Drivers

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**ABSTRACT**

Small-signal calculations show that the maximum nominal efficiency of a horn loudspeaker compression driver is 50% and the maximum true efficiency is 100%. Maximum efficiency occurs at the driver's resonance frequency. In the absence of driver mechanical losses, the maximum nominal efficiency occurs when the reflected acoustic load resistance equals the driver's voice-coil resistance and the maximum true efficiency occurs when the reflected acoustic load resistance is much higher that the driver's voice-coil resistance. To maximize the driver's broad-band true efficiency, the Bl force factor must be increased as much as possible, while jointly reducing moving mass, voice-coil inductance, mechanical losses, and front air-chamber volume. Higher compression ratios will raise high-frequency efficiency but may decrease mid-band efficiency. This paper will explore in detail the efficiency and design implications of both the nominal and true efficiency relationships including gain-bandwidth tradeoffs.

## 0. INTRODUCTION

The question of how high the efficiency of a loudspeaker or compression driver can go depends on how efficiency is defined. Typically two definitions of efficiency have been used: nominal efficiency and true efficiency. Commonly, compression driver efficiency is defined using the nominal definition which mimics the constant-voltage-drive pressure frequency response of the horn-driver combination [1-4]. Other authors sometimes use true efficiency[5-7].

The nominal efficiency of horn-driver systems is calculated by dividing the acoustic output power by the nominal electrical input power, which is in turn is calculated by squaring the input voltage and dividing by twice the resistance of the driver's voice coil. This method of calculating efficiency is essentially the same as the method used for direct-radiator systems [8–10] except that the input resistance for horn-driver systems is assumed to be twice the voice-coil resistance rather than equal to the voice-coil resistance, as it is for direct-radiator systems.

True efficiency however is calculated by dividing the acoustic output power by the true electrical input power. The true efficiency depends strongly on the input impedance of the compression driver. Higher input impedance directly results in higher true efficiency [10-12].

This paper will explore in detail the small-signal efficiency and compression driver design implications of both the nominal and true efficiency definitions using electric network analogs evaluated using the computer math program Maple [13].

This paper somewhat parallels the structure of an earlier paper by the author that explored the maximum efficiency of direct radiator loudspeakers [14]. That paper however only considered nominal efficiency.

Traditional compression driver design techniques based on nominal efficiency yield design values that fit narrow ranges. Raising or lowering these design parameters decreases the nominal efficiency.

Designs based on true efficiency however allow a much broader range of design parameters. For example, rather than the single value of Bl factor that maximizes nominal efficiency, the Bl can be raised essentially without limit to increase the true efficiency and bandwidth. Raising the Bl factor greatly raises the input impedance at and near the resonance of the compression driver and thus greatly improves the real efficiency. Higher Bl values also broaden the impedance peak and thus raises the real efficiency over a broad range.

The downside of the true-efficiency design process is that the constant-voltage-drive frequency response of the driver-horn combination is no longer flat. The response may be easily equalized however. This paper will explore those factors that maximize the true efficiency of the compression driver.

The paper is organized as follows: Section 1 defines the variables used in this paper. Section 2 describes the definition and assumptions of the efficiency definition used by the traditional design methods, based on nominal electrical input power. Section 3 describes true power transfer efficiency which is based on the actual electrical input power and the radiated acoustic output power. Section 4 derives the nominal and true efficiency equations for the general network block that is used to calculate efficiencies for the electric network models. Section 5 describes the mechanical and electrical models of the compression driver coupled to an infinite tube load. Section 6 derives the maximum efficiency conditions and efficiency-bandwidth products for several different simplifications of the compression driver's equivalent circuit. Section 7 illustrates the effect of changing a number of the parameters of a an example driver/horn system on the system's nominal and true frequency response, while section 8 concludes.

## 1. GLOSSARY OF SYMBOLS

$\alpha$ compression ratio or ratio of driver diaphragm area to horn throat (or tube) area ($=S_D/S_T$)

$B$ magnetic flux density in driver air gap

$\beta$ dimensionless ratio of electrical resistance representing the real part of the acoustic radiation load to the driver voice coil dc resistance

$$( = R_3 / R_1 = R_{ET} / R_E = \frac{S_T B^2 l^2}{R_E \rho_0 c {S_D}^2} )$$

$c$ velocity of sound in air ( = 343 m/s at 20° C)

$C_{AT}$ total acoustic compliance of driver and enclosure

$C_{MES}$ electrical capacitance representing driver moving mass including air-load mass ($= M_{MS} / B^2 l^2$)

$C_1$ pseudonym for $C_{MES}$

$E_{\text{in}}$ input voltage applied to terminals of loudspeaker, V rms

$E_{\text{out}}$ output voltage appearing across real part of radiation load in circuit model ($R_{ET}$)

$f$ natural frequency variable in Hz

$f_S$ resonance frequency of driver In free air

$H(s)$ voltage transfer function of general network block ( $= E_{\text{out}}(s) / E_{\text{in}}(s)$)

$I$ current In amps

$l$ length of voice-coil conductor in magnetic gap

$L_{CET}$ electrical inductance representing total system compliance of driver suspension and rear air chamber ($= C_{AT}B^2l^2 / S_D^2$)

$L_E$ electrical inductance of driver voice coil

$L_{FC}$ electrical inductance representing acoustic compliance of air in front air chamber

$M_{MS}$ mechanical moving mass of driver diaphragm assembly including air load

$P_A$ acoustic output power ( $= e_{out}^2 / R_{ET}$ in circuit model)

$P_E$ nominal electrical input power ( $= e_{in}^2 / R_E$ for direct-radiator systems and $= e_{in}^2 / 2R_E$ for horn-driver systems)

$Q$ ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)

$Q_{ES}$ Q of driver at $f_S$ considering electrical resistance $R_E$ only ( $= \frac{2\pi f_S R_E M_{MS}}{B^2 l^2}$ )

$R_{AS}$ acoustic resistance of driver suspension losses

$R_E$ dc resistance of driver voice coil

$R_{ET}$ electrical resistance representing the real part of the acoustic radiation load (here assumed constant and equal to be the acoustic load of an infinite tube) ( $= S_T B^2 l^2 / (\rho_0 c S_D^2) = B^2 l^2 / (\rho_0 c \alpha S_D)$ )

$R_{ES}$ electrical resistance representing driver mechanical losses ( $= B^2 l^2 / R_{MS}$ )

$R_L$ load resistance for general network block used to calculate efficiencies

$R_{MS}$ mechanical resistance of driver suspension losses ( $= \frac{S_D^2}{R_{AS}}$ )

$R_1$ pseudonym for $R_E$

$R_2$ pseudonym for $R_{ES}$

$R_3$ pseudonym for $R_{ET}$

$s$ complex frequency variable ( $= \sigma + j\omega$ )

$S_D$ effective surface area of driver diaphragm

$S_T$ area of horn throat or area of infinite tube load

$\tau$ dimensionless ratio of electrical resistance representing driver mechanical losses to the driver voice coil dc resistance ( $= R_2 / R_1 = R_{ES} / R_E$ )

$V_B$ volume of rear air chamber enclosing the back of the driver

$V_{FC}$ volume of front air chamber coupling the driver diaphragm to the horn throat or tube.

$\eta$ efficiency, (= power out divided by power in = $P_A/P_E$)

$\eta_{nom}$ nominal efficiency (acoustic power out divided by nominal electrical input power)

$\eta_{true}$ true efficiency (acoustic power out divided by true electrical input power)

$\eta_{max}$ maximum efficiency

$\rho_0$ density of air ( = 1.21 kg/m$^3$ )

$\omega$ radian frequency variable ( $= 2\pi f$ )

$\omega_{MN}$ radian rolloff frequency due to driver moving mass and driver voice-coil resistance ( $= 1/(R_1 C_1) = 1/(R_E C_{CMES})$ )

$\omega_N$ normalized frequency (= $\omega / \omega_{MN}$ for section 6.3 and = $\omega/\omega_0$ for section 6.4)

$\omega_0$ radian resonance frequency of driver mounted in rear air chamber ( $= 1/\sqrt{L_2 C_1}$ )

## 2. NOMINAL POWER TRANSFER EFFICIENCY

Typically, compression driver design techniques optimize the design for constant-voltage operation. This parallels the design methods for direct-radiator cone speakers that follow the teachings of Thiele and Small [8-9]. This is as it should be, because speakers and compression drivers are ordinarily driven by amplifiers with very-low output impedances which provide essentially constant-voltage operation regardless of loudspeaker impedance. Speakers and compression driver/horn combinations are also traditionally designed to have roughly flat acoustic frequency response when presented with a constant-voltage flat-response electrical input.

These operating conditions and assumptions drove the design techniques and particularly the definition of the electro-acoustic efficiency of a speaker system, the so-called nominal power transfer efficiency, which is defined as the acoustic power output divided by the nominal electrical input power.

### 2.1. Nominal Electrical Input Power

The nominal electrical input power to a compression driver is defined as the power delivered by the amplifier into a resistor having the same value as *twice* the driver's voice coil resistance (or sometimes defined as the compression driver's rated impedance or minimum impedance in the system's pass band). This is usually calculated by simply squaring the input voltage and dividing by twice the compression driver's voice coil resistance or rated impedance.

For purposes of calculating the nominal input power of a compression driver, note that nominal resistance is defined as twice the driver's voice-coil resistance rather than equal to the voice-coil resistance as it is for direct radiator systems. This is because horn-driver systems usually exhibit much higher motional impedance throughout their operating band due to their high-*Bl* low-mass low-loss design.

This definition of input power yields an efficiency vs. frequency response curve that mimics the SPL response curve you get when driving the system with a constant voltage source.

## 3. TRUE POWER TRANSFER EFFICIENCY

The true efficiency of a compression driver is defined as the acoustic power output divided by the true electrical input power.

### 3.1. True Electrical Input Power

The true electrical input power or average input power to a loudspeaker for steady-state sinusoidal operation is defined as the real part of the product of the rms input current and rms input voltage $\text{Real}(I \text{ x } E) = I \text{ x } E \text{ x } Cos(\theta)$, where $\theta$ is the angle between voltage and current and otherwise known as the power factor in power distribution systems. This definition of input power is not based on any fictitious power developed in a rated resistance but is the actual power drawn by the speaker. Note that if the loudspeaker load impedance is essentially reactive, its actual or true power drawn from the source amplifier is very low, regardless of its impedance magnitude.

## 4. COMPUTATION OF EFFICIENCY FOR GENERAL NETWORK BLOCK

This section develops the nominal and true power transfer ratio or efficiency equations for a general electric network block. These equations will be an aid in calculating the actual nominal and true efficiency of the electric network models that follow in this paper.

Figure 1 shows the general electric network block with load resistance $R_L$ connected at the output, and voltage $e_{in}$ applied to the input. The applied voltage causes current $I_{in}$ to flow into the block due to the block's input impedance $Z_{in.}$ The nominal and true efficiency expressions for this system will be developed in the next two subsections.

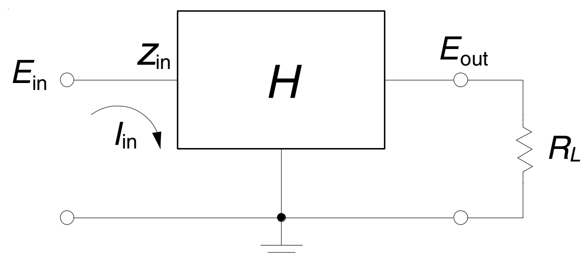



Fig. 1. General electric network block with load $R_L$ driven by voltage source $E_{in}$. An analysis of this model is used as an aid in calculating the nominal and true efficiencies of the compression driver's electric network model.

### 4.1. Nominal Power Input

The nominal input power of the general network block of Fig. 1 is calculated by squaring the rms input voltage and dividing by twice the voice-coil resistance of the compression driver (by definition)

$$P_{nom} = \frac{|E_{in}|^2}{2R_E} \tag{1}$$

### 4.2. True Power Input

The true input power of the general network block of Fig. 1 is computed by squaring the rms input current and multiplying by the real part of the block's input impedance

$$\begin{aligned} P_{true} &= |I_{\text{in}}|^2 \Re(Z_{\text{in}}) \\ &= \left|\frac{E_{\text{in}}}{Z_{\text{in}}}\right|^2 \Re(Z_{\text{in}}) \\ &= \frac{|E_{\text{in}}|^2}{|Z_{\text{in}}|^2} \Re(Z_{\text{in}}) \end{aligned} \tag{2}$$

### 4.3. True Power Output

The output power of the general network block of Fig. 1 is developed in the load resistance $R_L$ and is given by

$$P_{\text{out}} = \frac{|E_{\text{out}}|^2}{R_L} \quad (3)$$

Equation 3 can be rewritten in a form that includes the voltage-transfer function of the block itself $H(j\omega)$ by multiplying and dividing the numerator by *E*in:

$$P_{\text{out}} = \frac{\left|E_{\text{in}}\left(\frac{E_{\text{out}}}{E_{\text{in}}}\right)\right|^2}{R_L} = \frac{|E_{\text{in}}|^2\left|\frac{E_{\text{out}}}{E_{\text{in}}}\right|^2}{R_L} = \frac{|E_{\text{in}}|^2\,|H(j\omega)|^2}{R_L} \quad (4)$$

### 4.4. Nominal Power Transfer Efficiency

The nominal efficiency is calculated by dividing Eq. (3) by Eq. (1) as follows:

$$\eta_{nom} = \frac{P_{\text{out}}}{P_{\text{nom}}} = \frac{\frac{|E_{\text{out}}|^2}{R_L}}{\frac{|E_{in}|^2}{2R_E}} = \frac{2R_E}{R_L}\left|\frac{E_{\text{out}}}{E_{in}}\right|^2 = \frac{2R_E}{R_L}\,|H(j\omega)|^2 \quad (5)$$

Equation (5) indicates that the nominal efficiency is just dependent on the ratio of the driver's voice-coil resistance to the block's load resistance and the square of the voltage transfer function of the block.

### 4.5. True Power Transfer Efficiency

The true efficiency is calculated by dividing Eq. (3) by Eq. (2) as follows:

$$\eta_{\text{true}} = \frac{P_{\text{out}}}{P_{\text{true}}} = \frac{\frac{|E_{\text{out}}|^2}{R_L}}{\frac{|E_{\text{in}}|^2}{|Z_{\text{in}}|^2}\Re(Z_{\text{in}})} = \frac{|Z_{\text{in}}|^2}{R_L\Re(Z_{\text{in}})}\left|\frac{E_{\text{out}}}{E_{\text{in}}}\right|^2 = \frac{|Z_{\text{in}}|^2}{R_L\Re(Z_{\text{in}})}\,|H(j\omega)|^2 \quad (6)$$

This interesting result shows that the true efficiency is directly proportional to magnitude of the input impedance squared $|Z_{\text{in}}|^2$. This implies that anything that can be done to increase the input impedance, directly results in higher true efficiency. This is behavior is quite intuitive but it nice to see that the equations predict this. What's not quite as intuitive, is that the true efficiency depends inversely on the load resistance value.

## 5. COMPRESSION DRIVER MODEL

### 5.1. Mechanical Model

Figure 2 shows a depiction of a direct-radiator cone loudspeaker used as a compression driver feeding an infinite tube load through in intermediate coupling volume $V_{FC}$. The rear of the speaker is enclosed with a rear volume $V_B$. The ratio between the speaker's diaphragm area $S_D$ and the infinite tubes area (or horn's throat area) $S_T$ is known as the compression ratio $\alpha$ $(= S_D / S_T)$. A large diaphragm driving a small throat results in a high compression ratio.

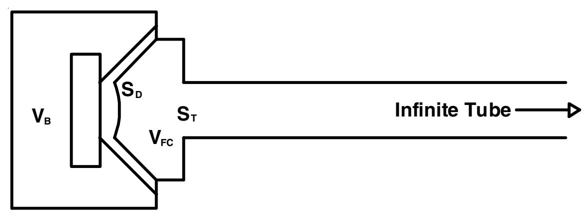



Fig. 2. Depiction of a cone loudspeaker driving an infinite tube through a coupling volume (front air chamber). The loudspeaker has an effective piston area $S_D$, drives an infinite tube of area $S_T$, with a front chamber volume of $V_{FC}$ and a rear chamber volume of $V_B$. The compression ratio $\alpha$ is the ratio between the driver's piston area and the tube's or equivalently the horn's throat area (= $S_T$/$S_D$). Traditionally, a "compression driver" includes not just the speaker or motor mechanism but the front cavity volume and any associated phasing plug leading up to an exit which couples to the horn throat.

### 5.2. Electrical Equivalent Circuit

Figure 3 shows the simplified lumped electrical equivalent circuit of the model of Fig. 2.

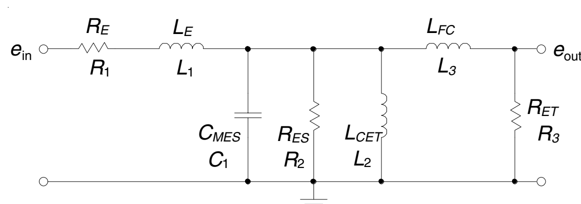



Fig. 3. Simplified lumped electrical equivalent circuit of the compression driver driving an infinite tube depicted in Fig.2. Refer to the glossary of symbols for explanation of the component values. The resistor representing the tube acoustic load resistance appears as resistor $R_{ET}$.

## 6. MAXIMUM EFFICIENCY

In this section, various simplifications of the circuit of Fig. 3 will be analyzed to determine the maximum efficiency. Section 6.1 considers only the driver's voice-coil resistance coupled directly to the resistance representing the acoustic load of the infinite tube. Section 6.2 adds a resistor representing the driver's mechanical losses to the circuit of section 6.1. Section 6.2 adds a capacitor representing the drivers moving mass to the circuit of section 6.1 and calculates an efficiency-bandwidth product. Section 6.4 considers the complete circuit of Fig. 3 but excludes voice-coil inductance and the effect of front-cavity compliance. The equations for the nominal and true efficiency frequency responses for the complete circuit of Fig. 3 are displayed in Section 6.5

### 6.1. Absolute Maximum Efficiency: Include Only Voice-Coil Resistance and Acoustic Load Resistance

"It is interesting to examine the efficiency that might be achieved if all the system reactances could be neglected and power could be delivered directly from the source to the radiation load. Such a situation is illustrated in Fig. 4, where the intervening reactances have been removed and the circuit thus simplified to a resistive voltage divider. While this situation is not realizable in practice, it does enable the calculation of an efficiency limit which certainly cannot be exceeded by any real system."[1]

#### *6.1.1. Simplified Circuit*

The model for this situation is the simplest possible case and just includes the driver dc voice coil resistance ($R_1$ = $R_E$) connected directly to the radiation load ($R_3 = R_{ET} = S_T B^2 l^2 / (\rho_0 c S_D{}^2)$). The driver diaphragm area $S_D$, horn throat (tube) area $S_T$, and *Bl* product are the only system parameters that can be adjusted to set the relative size of the reflected radiation resistance as compared to $R_E$. Adjusting these parameters raises and lowers the efficiency, but as will be shown, the efficiency can only be raised to 50% for nominal efficiency, but can be raised to 100% for true efficiency.

[1] These are not the author's words but the words of R. H. Small in a suggested revision when he reviewed my paper concerning the maximum efficiency of direct radiators back in 1992 [14]. This paper was recommended for publication but due to the my laziness in performing the suggested revisions has not been published.

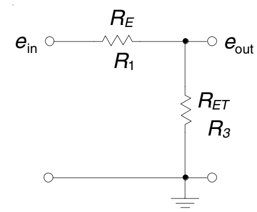



Fig. 4. Analogous circuit of Fig. 3 with all system reactances and extraneous losses removed. Only the driver's dc resistance $R_E$ and the acoustic radiation resistance $R_{ET}$ remain. This is equivalent to the impossible situation of no voice-coil inductance, infinite system compliance, zero moving mass, no mechanical losses, and no front-air chamber.

#### *6.1.2. Nominal Efficiency*

By inspection, the voltage transfer function for this circuit is

$$H(j\omega) = \frac{R_3}{R_1 + R_3} \tag{7}$$

This can be combined with Eq. (5) along with the substitutions $R_e = R_1$ and $R_L = R_3$ to yield

$$\eta_{\text{nom}} = \frac{2 R_1 R_3}{(R_1 + R_3)^2} \tag{8}$$

This can be further simplified with the normalization variable $\beta = R_3 / R_1$, which is the ratio between the acoustic load resistance and the drivers dc resistance, yielding

$$\eta_{nom} = \frac{2\beta}{(\beta + 1)^2} \tag{9}$$

Figure 5 shows a plot of this function along with a plot of the true efficiency which is derived in the next section. The plot clearly shows that the nominal efficiency reaches a maximum of 0.5 (50%) at $\beta = 1$.

The normalization variable $\beta$ can be converted to driver-horn (tube) mechanical-acoustical variables as follows

$$\beta = \frac{R_3}{R_1} = \frac{\left(\frac{1}{\rho_0 c}\right) \frac{S_T}{S_D{}^2} B^2 l^2}{R_E} = \frac{S_T B^2 l^2}{\rho_0 c R_E S_D{}^2} \tag{10}$$

If Eq. (10) is equated to 1, the condition for maximum nominal efficiency, the horn-throat area $S_T$ that maximizes nominal efficiency can be calculated:

$$S_T = \frac{\rho_0 c R_E S_D{}^2}{B^2 l^2} \qquad (11)$$

This is recognized as being the same expression calculated previously in 1977 [1]. Note that if the throat area is increased or decreased, the nominal efficiency will decrease.

### 6.1.3. *True Efficiency*

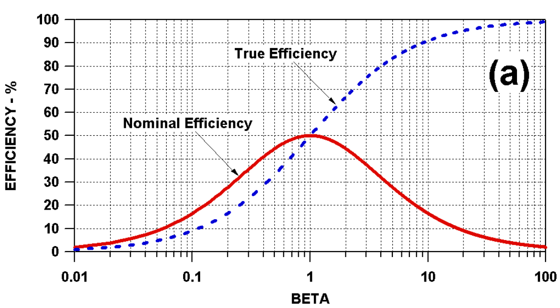
A parallel computation to Eqs. 7-9 yields the following

$$\eta_{true} = \frac{R_3}{R_3 + R_1} \qquad (12)$$

and

$$\eta_{true} = \frac{\beta}{\beta + 1} \qquad (13)$$

Figure 5 shows a plot of this function along with a plot of the nominal efficiency which was derived in the previous section. The plot clearly indicates that the true efficiency increases directly with $\beta$. Very high values of $\beta$ result in true efficiencies approaching 100%. The next section will show however, that this is not true when driver mechanical losses are included.



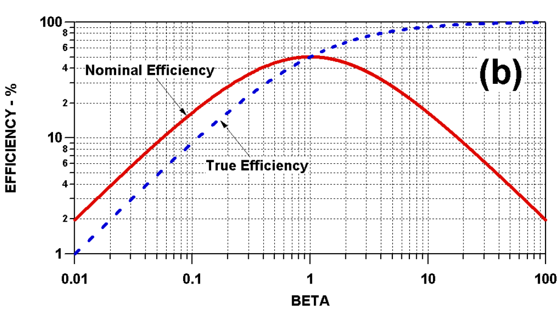




Fig. 5. Graphs depicting the nominal and true efficiency of the simplified circuit of Fig. 4 as a function of the normalized variable $\beta$ which is the ratio between the acoustic load resistance and the drivers dc resistance ($\beta = R_3/R_1$). (a) Linear vertical efficiency scale. (b) Log vertical efficiency scale. This represents the theoretical maximum efficiency that the compression driver can exhibit if all system reactances and losses are neglected. Note that the nominal efficiency reaches a maximum of 50% at $\beta$ = 1, but that the true efficiency continues to increase and approaches 100% as $\beta$ is increased.

Equation (13) can be recast in a form that includes the system parameters

$$\eta_{true} = \frac{1}{1 + \dfrac{\rho_0 c R_E S_D{}^2}{S_T B^2 l^2}} \qquad (14)$$

Interestingly, Eq. (14) indicates that the true efficiency can be increased without limit by raising $Bl$ or throat area $S_T$ or by decreasing voice coil resistance $R_E$ or diaphragm area $S_D$. Unlike the nominal efficiency, there is not a specific value of these parameters that maximizes the efficiency. They may be changed without limit to increase the efficiency towards 100% (Note! This last statement is true only in the absence of mechanical losses. The next section will show that a design that includes finite losses will also exhibit an optimum value of $\beta$ that maximizes true efficiency and results in a relationship similar to Eq. (11).)

## 6.2. Efficiency: Include Only Voice-Coil Resistance, Acoustic Load Resistance, and Mechanical Losses

This section adds mechanical losses to the circuit of Fig. 4 by the addition of $R_{ES}$ in parallel with $R_{ET}$.

### 6.2.1. *Simplified Circuit*

This circuit is shown in Fig. 6.

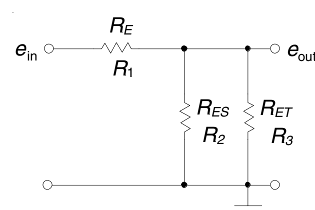



Fig. 6. Analogous circuit of Fig. 4 but with the resistor representing driver mechanical resistance added. Only the driver's dc resistance and the acoustic radiation resistance remain. The addition of the driver's mechanical losses reduces the efficiency of the driver.

### 6.2.2. *Nominal Efficiency*

In terms of the circuit components the calculation shows that the nominal efficiency is given by

$$\eta_{nom} = \frac{2R_1R_2^2R_3}{(R_1R_2 + R_1R_3 + R_2R_3)^2} \quad (15)$$

This can be converted to normalized parameters with the substitutions of $\beta = R_3/R_1$ , and $\tau = R_2/R_1$ .

$$\eta_{nom} = \frac{2\beta\tau^2}{(\beta+\beta\tau+\tau)^2} = \frac{2\beta}{\left(\frac{\beta}{\tau}+\beta+1\right)^2} \quad (16)$$

From the last form of Eq. (16), it is clear that when the mechanical losses go to zero ( $\tau \to \infty$ or $R_{ES} \to \infty$ ), the limit of Eq. (16) is equal to Eq. (9), the nominal efficiency in the absence of driver mechanical losses.

Figure 7 shows plots of the nominal efficiency function of Eq. (16) as a function of $\beta$ for several values of $\tau$ in the range of 0.1 to 10,000 in step ratios of 10. This graph also includes a plot of the true efficiency which is computed in the next section.

Note that for high mechanical losses ( $\tau < 1$ ) the efficiency is greatly attenuated no matter what the value of $\beta$ and that the location of the peak efficiency drops below $\beta$ =1. For lower mechanical losses ( $\tau \geq 1$ ), the efficiency increases only to 50% while the location of the peak stays nears $\beta$ = 1. The following Eqs. (17) and (18) confirm this behavior.

Calculation shows that the peak nominal efficiency occurs at

$$\beta = \frac{\tau}{\tau+1} \quad (17)$$

where the nominal efficiency is

$$\eta_{nom} = \frac{\tau}{2(\tau+1)} \quad (18)$$

### 6.2.3. *True Efficiency*

Likewise, the true efficiency of Fig. 6 is found to be

$$\eta_{true} = \frac{R_2^2R_3}{(R_2+R_3)(R_1R_2 + R_1R_3 + R_2R_2)} \quad (19)$$

This in turn can be converted to the normalized parameters $\beta$ and $\tau$ .

$$\eta_{true} = \frac{\beta\tau^2}{(\beta+\tau)(\beta+\beta\tau+\tau)} = \frac{\beta}{\left(\frac{\beta}{\tau}+1\right)\left(\frac{\beta}{\tau}+\beta+1\right)} \quad (20)$$

From the last form of Eq. (20), it is clear that the limit when $\tau$ is raised without limit ( $\tau \to \infty$ or $R_{ES} \to \infty$ ) is equal to Eq. (13), the true efficiency in the absence of driver mechanical losses.

Figure 7 shows plots of the true efficiency function of Eq. (20) as a function of $\beta$ for several values of $\tau$ in the range of 0.1 to 10,000 in step ratios of 10.

Note that for finite values of $\tau$ , the true efficiency reaches an intermediate peak and then falls at higher values of $\beta$ . As with the nominal efficiency, the true efficiency is greatly attenuated for $\tau < 1$ , but increases to higher values for $\tau \geq 1$ .

Calculation shows that the peak true efficiency occurs at

$$\beta = \frac{\tau}{\sqrt{\tau+1}} \quad (21)$$

where the true efficiency is

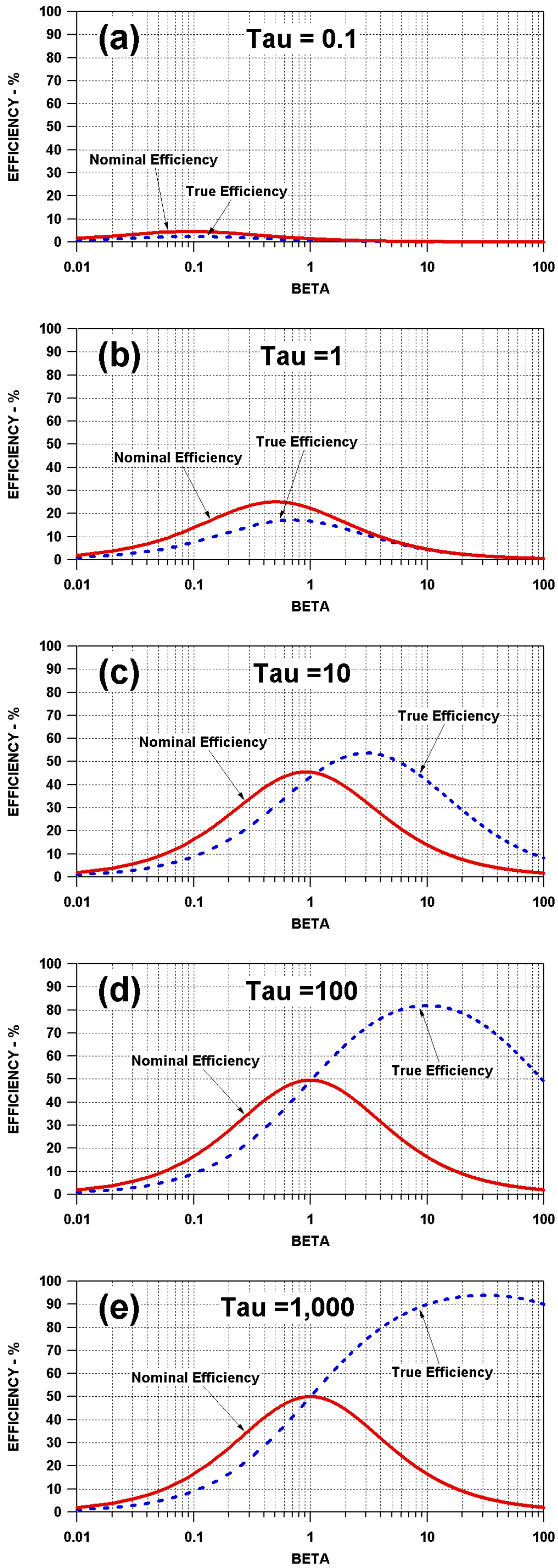
$$\eta_{true} = \frac{\tau\sqrt{\tau+1}}{(\sqrt{\tau+1}+1)(\tau+1+\sqrt{\tau+1})} \quad (22)$$

This implies that any real world compression driver design that includes finite mechanical losses, has a specific value of $\beta$ that maximizes the true efficiency.

When the normalization variable $\beta$ is converted to mechanical-acoustical variables Eq. (10), the true efficiency is maximized for the following condition

$$\beta = \frac{S_T B^2 l^2}{\rho_0 c R_E S_D^{\ 2}} = \frac{\tau}{\sqrt{\tau+1}} \quad (23)$$

With the knowledge that
$\tau = R_2/R_1 = R_{ES}/R_E = R_{AS}B^2l^2/(R_E S_D^{\ 2})$ and Eq. (23), any parameter can be calculated in terms of the remaining parameters to maximize the efficiency.



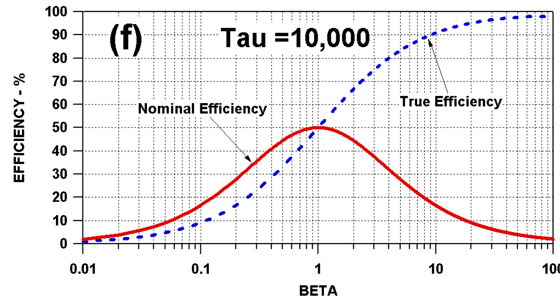




Fig. 7. Graphs showing the nominal and true efficiency of the simplified circuit of Fig. 6 as a function of the normalized variable $\beta$ and various values of the normalized loss resistance $\tau$ (=$R_2/R_1$). (a) $\tau$ =0.1. (b) $\tau$ =1. (c) $\tau$ =10. (d) $\tau$ =100. (e) $\tau$ =1,000. (f) $\tau$ =10,000. High values of $\tau$ correspond to low driver mechanical losses. The main effect of high driver losses ($\tau < 5$) is the reduction of efficiency over the whole range of $\beta$ s. The other main effect is the reduction of true efficiency for high $\beta$ values. This means that only one $\beta$ value maximizes efficiency for a specific value of $\tau$ .

## 6.3. Efficiency: Include Only Voice-Coil Resistance, Acoustic Load Resistance, and Moving Mass

This section adds driver moving mass to the circuit of Fig. 4 by the addition of $C_{ES}$ in parallel with $R_{ET}$. Addition of moving mass causes a first-order rolloff which reduces the efficiency at high frequencies.

### *6.3.1. Simplified Circuit*

This circuit is shown in Fig. 8.

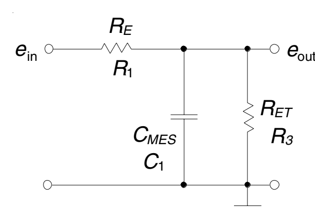



Fig. 8. Analogous circuit of Fig. 4 but with the capacitor representing driver moving mass added. The addition of the driver's moving mass reduces the efficiency of the driver at high frequencies. The mass causes a first-order efficiency rolloff above a certain frequency.

### *6.3.2. Nominal Efficiency*

Analysis of Fig. 8 reveals that the nominal efficiency is given by

$$\eta_{nom}(\omega) = \frac{2R_1R_3}{R_1^{\,2}R_3^{\,2}C_1^{\,2}\omega^2 + (R_1+R_3)^2} = \frac{\dfrac{2}{R_1R_3C_1^{\,2}}}{\omega^2 + \dfrac{(R_1+R_3)^2}{R_1^{\,2}R_3^{\,2}C_1^{\,2}}} = \frac{\dfrac{2}{R_1R_3C_1^{\,2}}}{\omega^2 + \dfrac{1}{\left(\dfrac{R_1R_3}{R_1+R_3}C_1\right)^2}} \tag{24}$$

This can be further simplified with the normalization variables $\beta = R_3/R_1$, $\omega_{MN} = 1/(R_1C_1)$, and $\omega_N = \omega/\omega_{MN}$ yielding

$$\eta_{nom}(\omega) = \frac{2\beta}{\omega_N^{\,2}\beta^2 + (\beta+1)^2} = \frac{\dfrac{2}{\beta}}{\omega_N^{\,2} + \left(\dfrac{\beta+1}{\beta}\right)^2} \tag{25}$$

Equation (25) indicates that the nominal efficiency rolloff corner frequency where the efficiency drops by one half is given by

$$\omega_N = \frac{\beta+1}{\beta} \tag{26}$$

At frequencies much below $\omega_N$ ($\omega_N << 1$) the nominal efficiency rises to a maximum of

$$\eta_{nom} = \frac{2\beta}{(\beta+1)^2} \tag{27}$$

This is of course equal to the maximum of the simplified circuit of Fig. 4 and given by Eq. (9).

Figure 9 shows plots of the nominal efficiency frequency response of Eq. (25) for several values of $\beta$ in the range of 0.01 to 100 with a step ratio of 10. Also plotted is the true efficiency derived in the next section. Note that low values of $\beta$ ($\beta < 1$) extend the high-frequency range of both nominal and true efficiencies, but that high $\beta$ values ($\beta >> 1$) cause no decrease of the nominal efficiency corner frequency below $\omega_N = 1$.

### *6.3.3. True Efficiency*

Analysis of Fig. 8 reveals that the true efficiency is

$$\eta_{true}(\omega) = \frac{R_3}{R_1R_3^{\,2}C_1^{\,2}\omega^2 + R_1 + R_3} \tag{28}$$

This can be converted to normalized form by the normalization variables $\beta$ and $\omega_N$

$$\eta_{true}(\omega) = \frac{\beta}{\beta^2\omega_N^{\,2} + \beta + 1} = \frac{\dfrac{1}{\beta}}{\omega_N^{\,2} + \dfrac{\beta+1}{\beta^2}} \tag{29}$$

Equation (27) shows that the rolloff corner frequency for true efficiency is given by

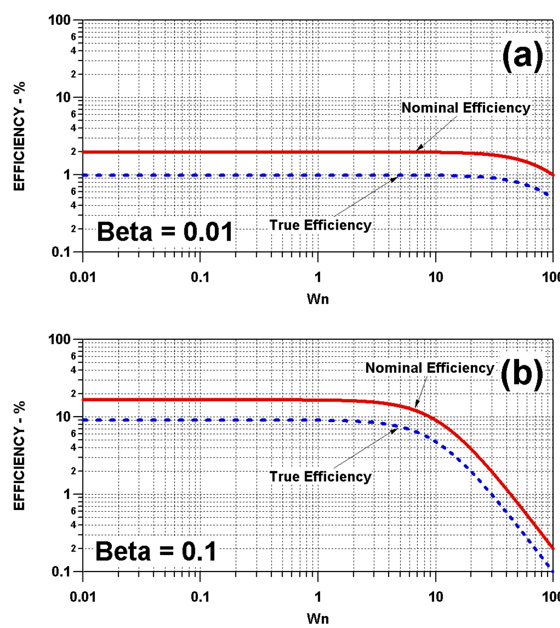
$$\omega_N = \frac{\sqrt{\beta+1}}{\beta} \tag{30}$$

At frequencies much below $\omega_N$ ($\omega_N << 1$) the true efficiency rises to a maximum of

$$\eta_{nom} = \frac{\beta}{\beta+1} \tag{31}$$

This is of course equal to the maximum of the simplified circuit of Fig. 4 and given by Eq. (13).

Figure 9 shows plots of the true efficiency and nominal efficiency for several values of $\beta$. Note that in contrast to the nominal efficiency rolloff frequency, high values of beta do cause a reduction of the true efficiency rolloff frequency.



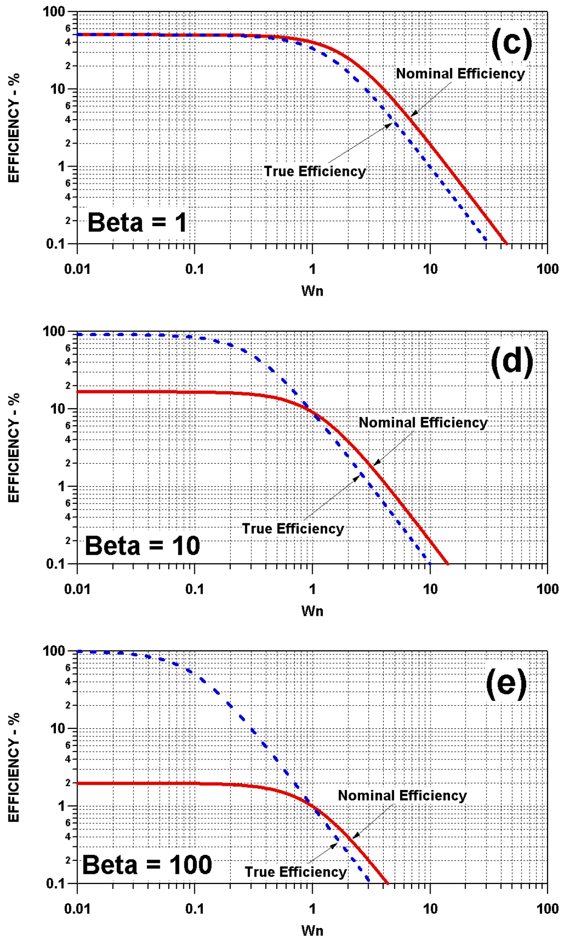



Fig. 9. Graphs showing the nominal and true efficiency frequency responses calculated from the circuit of Fig. 8 for various values of $\beta$. The frequency responses are plotted against the normalized frequency variable $\omega_N$ which is the frequency where the driver's moving mass reactance equals the driver's voice-coil resistance. (a) $\beta$ = 0.01. (b) $\beta$ = 0. 1. (c) $\beta$ = 1. (d) $\beta$ = 10. (e) $\beta$ = 100. True efficiency dashed line. Nominal efficiency solid line. Note that the LF ( $\omega_N$ << 1) true efficiency increases with $\beta$, but that the mass roll-off frequency decreases. Also note that the LF nominal efficiency reaches a maximum at $\beta$ = 1, but that the mass roll-off corner frequency stays roughly the same at $\omega_N$ = 1 for $\beta$ > 10.

### *6.3.4. Efficiency-Bandwidth Product*

The next two sections calculate the efficiency-bandwidth (EB) product for the nominal and true efficiencies for the circuit of Fig. 8.

*6.3.4.1. Nominal Efficiency*

The bandwidth of the nominal efficiency equation Eq. (25) is given by Eq. (26) which is repeated here

$$\omega_N = \frac{\beta+1}{\beta} = \Delta\omega \tag{32}$$

This is the desired bandwidth value $\Delta\omega$. The efficiency-bandwidth product is then formed by the product of Eq. (27) and Eq. (32)

$$\begin{aligned} \mathrm{EB_{nom}} &= \max(\eta_{nom})\Delta\omega \\ &= \frac{2\beta}{(\beta+1)^2}\,\frac{\beta+1}{\beta} \\ &= \frac{2}{\beta+1} \end{aligned} \tag{33}$$

*6.3.4.2. True Efficiency*

The bandwidth of the true efficiency equation Eq. (29) is given by Eq. (30) which is repeated here

$$\omega_N = \frac{\sqrt{\beta+1}}{\beta} = \Delta\omega \tag{34}$$

which is the desired bandwidth value $\Delta\omega$. The efficiency-bandwidth product is then formed by the product of Eq. (31) and Eq. (34)

$$\begin{aligned} \mathrm{EB_{nom}} &= \max(\eta_{nom})\Delta\omega \\ &= \frac{\beta}{\beta+1}\,\frac{\sqrt{\beta+1}}{\beta} \\ &= \frac{1}{\sqrt{\beta+1}} \end{aligned} \tag{35}$$

The nominal and true efficiency-bandwidth products are plotted as a function of $\beta$ in Fig. 10. Both EB products are equal at $\beta = 3$ with the nominal efficiency EB product higher below and the true EB product higher above.

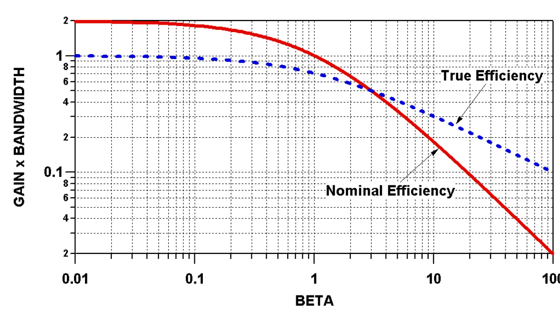



Fig. 10. Plots of the efficiency-bandwidth product versus $\beta$ for the nominal and true efficiencies calculated for the circuit of Fig. 8, which includes only the effect of the driver's moving mass. Note that the gain-bandwidth product is maximized for $\beta$ < 1. Note also that the nominal efficiency efficiency-bandwidth

product rolls-off at twice the rate of the true efficiency gain-bandwidth product above $\beta = 3$.

### *6.3.5. Moving Mass Rolloff Frequency in Terms of Thiele/Small Parameters*

An interesting relationship for the upper-mass rolloff frequency can be derived using the direct-radiator Thiele/Small driver parameters.

Equation (26) can be solved for the half-power corner frequency which is the desired upper-mass rolloff corner:

$$\begin{aligned}\omega = \omega_{HM} &= \omega_{MN}\omega_N \\ &= \frac{1}{R_1 C_1}\frac{\beta+1}{\beta} \\ &= \left(\frac{1}{\frac{\beta}{\beta+1}R_1 C_1}\right)\end{aligned} \tag{36}$$

The component values $R_1$ and $C_1$ can then be converted into their electric network equivalent values: $R_1 = R_E$ and $C_1 = \frac{M_{MS}}{B^2 l^2}$, along with $\omega_{HM} = 2\pi f_{HM}$ yielding

$$f_{HM} = \frac{B^2 l^2}{2\pi\left(\frac{\beta}{\beta+1}R_E M_{MS}\right)}. \tag{37}$$

We know that the electrical quality factor $Q_{ES}$ of the driver is given by

$$Q_{ES} = \frac{2\pi f_S R_E M_{MS}}{B^2 l^2} \tag{38}$$

from which Eq. (37) may be converted to

$$\begin{aligned}f_{HM} &= \frac{1}{\left(\frac{\beta}{\beta+1}\right)}\frac{f_S}{Q_{ES}} \\ &= \frac{1}{\eta_{true}}\frac{f_S}{Q_{ES}}\end{aligned} \tag{39}$$

which is the desired result.

Equation (39) clearly shows the inverse relationship between the driver's true efficiency $\eta_{true}$ and the driver's mass rolloff frequency when driving the tube load. The equation also shows that the mass rolloff frequency can be increased by raising the drivers free-air resonance $f_S$ or decreasing the electrical quality factor $Q_{ES}$.

## 6.4. Efficiency: Include Only Voice-Coil Resistance, Acoustic Load Resistance, Moving Mass, System Total Compliance, and Mechanical Resistance

In this section, only the driver's voice coil inductance and front cavity compliance are neglected. Fig. 11 shows this circuit. To minimize transcribing errors due to equation complexities, this section lists some of the equations in raw Maple output as captured on screen.

### *6.4.1. Simplified Electrical Circuit*

Figure 11 shows the circuit for this situation. The inductances representing voice-coil inductance and front-cavity compliance have been removed. The circuit forms a pure second-order band-pass filter whose center frequency is the resonance of the mass-compliance of the compression driver.

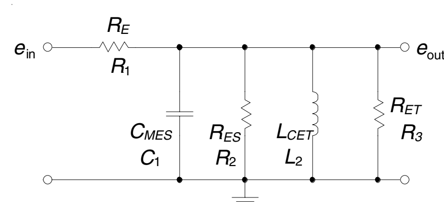



Fig. 11. Analogous circuit of Fig. 3 but with the circuit components representing driver voice-coil inductance and front-cavity inductance removed.

### *6.4.2. Nominal Efficiency*

Analysis of Fig. 11 reveals that the nominal efficiency is given by the following equation which is shown in raw Maple output. Note that variable subscripting has been suppressed. The denominator of this expression is 4th order in $\omega$.

$\eta_{nom}$ =

```
> NominalEfficiencyCollected;
```

$$\left(2\,L2^2\,\omega^2\,R3\,R2^2\,R1\right)\Big/\Big(R3^2\,R2^2\,R1^2\,\omega^4\,C1^2\,L2^2 + (2\,R1\,L2^2\,R2\,R3^2 + R2^2\,R3^2\,L2^2 - 2\,R3^2\,R2^2\,R1^2\,C1\,L2 + R1^2\,L2^2\,R2^2 + R3^2\,R1^2\,L2^2 + 2\,R3\,R1\,L2^2\,R2^2 + 2\,R3\,R2\,R1^2\,L2^2)\,\omega^2 + R3^2\,R2^2\,R1^2\Big) \tag{40}$$

Equation 40 may be simplified by introducing the normalization variables: $\omega_0 = 1\big/\sqrt{L_2 C_1}$, $\omega_n = \omega/\omega_0$, $\tau = R_2/R_1$, $\omega_{MN} = 1/R_1 C_1$, and $\beta = R_3/R_1$ yielding

$\eta_{nom}$ =

`> NominalEfficiency;`

$$\left(2\,\beta\,\tau^2\,Wn^2\,Wmn^2\right) \Big/ \Big(\tau^2\,\beta^2\,Wn^4 + \left(-2\,\tau^2\,\beta^2 + Wmn^2\,\tau^2\,\beta^2 + Wmn^2\,\beta^2 + Wmn^2\,\tau^2 + 2\,Wmn^2\,\tau^2\,\beta + 2\,Wmn^2\,\tau\,\beta + 2\,Wmn^2\,\beta^2\,\tau\right) Wn^2 + \tau^2\,\beta^2\Big) \qquad (41)$$

Figures 12 – 14 show nominal-efficiency frequency responses calculated from Eq. (41) (see section 6.4.4 for descriptions).

### *6.4.3. True Efficiency*

Analysis of Fig. 11 reveals that the true efficiency is given by the following equation which is shown in raw Maple output. Note that variable subscripting has been suppressed. The denominator of this expression is 4th order in $\omega$.

$\eta_{true}$ =

`> TrueEfficiencyCollected;`

$$\left(L2^2\,\omega^2\,R3\,R2^2\right) \Big/ \Big(R1\,R2^2\,R3^2\,\omega^4\,C1^2\,L2^2 + (R1\,L2^2\,R3^2 + R1\,L2^2\,R2^2 + L2^2\,R2^2\,R3 - 2\,R1\,R2^2\,R3^2\,C1\,L2 + 2\,R1\,L2^2\,R2\,R3 + L2^2\,R2\,R3^2)\,\omega^2 + R1\,R2^2\,R3^2\Big) \qquad (42)$$

As with Eq. 40, Eq. (42) may be simplified by introducing the same normalization variables:

`> TrueEfficiency;`

$$\left(\beta\,\tau^2\,Wn^2\,Wmn^2\right) \Big/ \Big(\tau^2\,\beta^2\,Wn^4 + \left(Wmn^2\,\beta^2\,\tau + Wmn^2\,\beta^2 + Wmn^2\,\tau^2 - 2\,\tau^2\,\beta^2 + 2\,Wmn^2\,\tau\,\beta + Wmn^2\,\tau^2\,\beta\right) Wn^2 + \tau^2\,\beta^2\Big) \qquad (43)$$

Figures 12 – 14 show true-efficiency frequency responses calculated from Eq. (43) (see next section for descriptions).

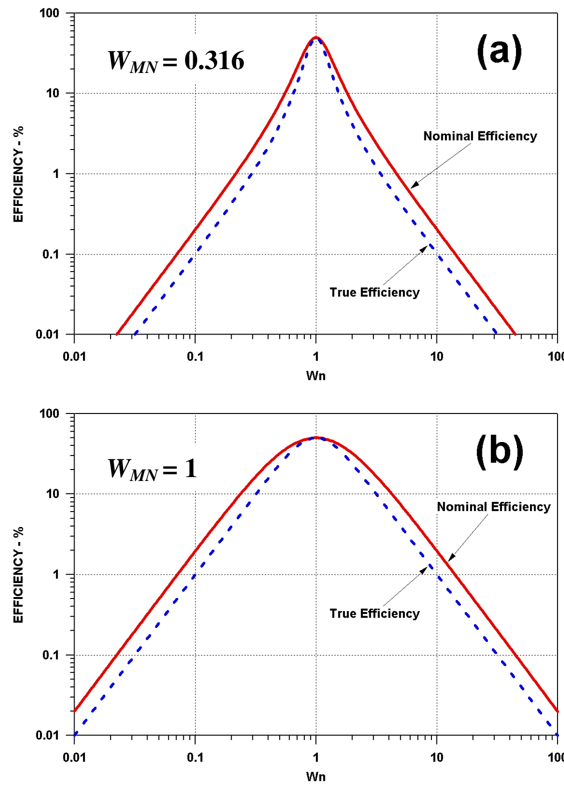
### *6.4.4. Efficiency vs. Frequency*

This section illustrates several efficiency frequency responses calculated from Eqs. (41) and (43). All the responses are normalized to the compression drivers resonance frequency ($\omega_n = \omega/\omega_0$). The next three subsections show responses where $\omega_{MN}$, $\beta$, and $\tau$ are varied. All responses are second-order bandpass.

#### *6.4.4.1. Vary Moving-Mass Rolloff Frequency( $\omega_{MN}$ )*

Figure 12 shows efficiency frequency responses of the circuit Fig. 11 while varying the driver's moving mass over a decade and a half range in half-decade steps ($\omega_{MN}$ = 0.316, 1, 3.16, and 10). Here $\beta = 1$, and $\tau = \infty$.

Note that as $\omega_{MN}$ increases, the width of the band-pass response also increases with the maximum efficiency at $\omega_N = 0$ remaining constant at 50% (as it should with zero mechanical losses and $\beta = 1$). Note that the true efficiency responses are slightly narrower than the nominal efficiency responses.



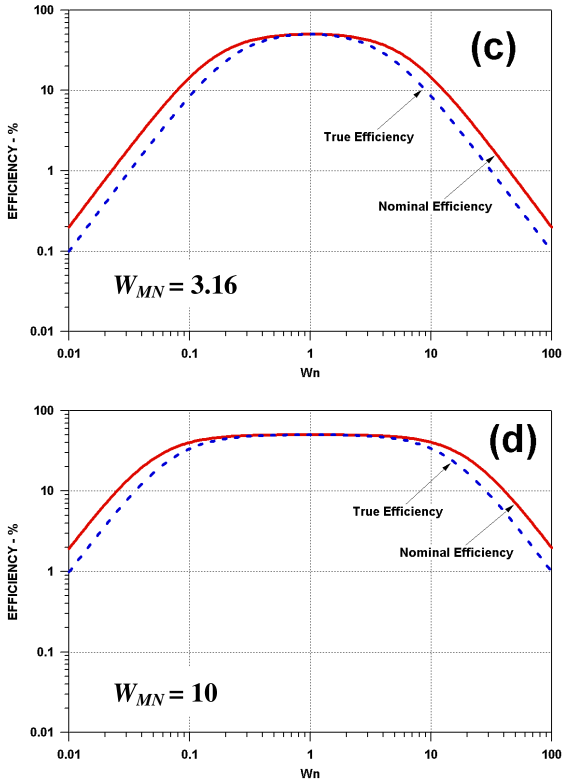
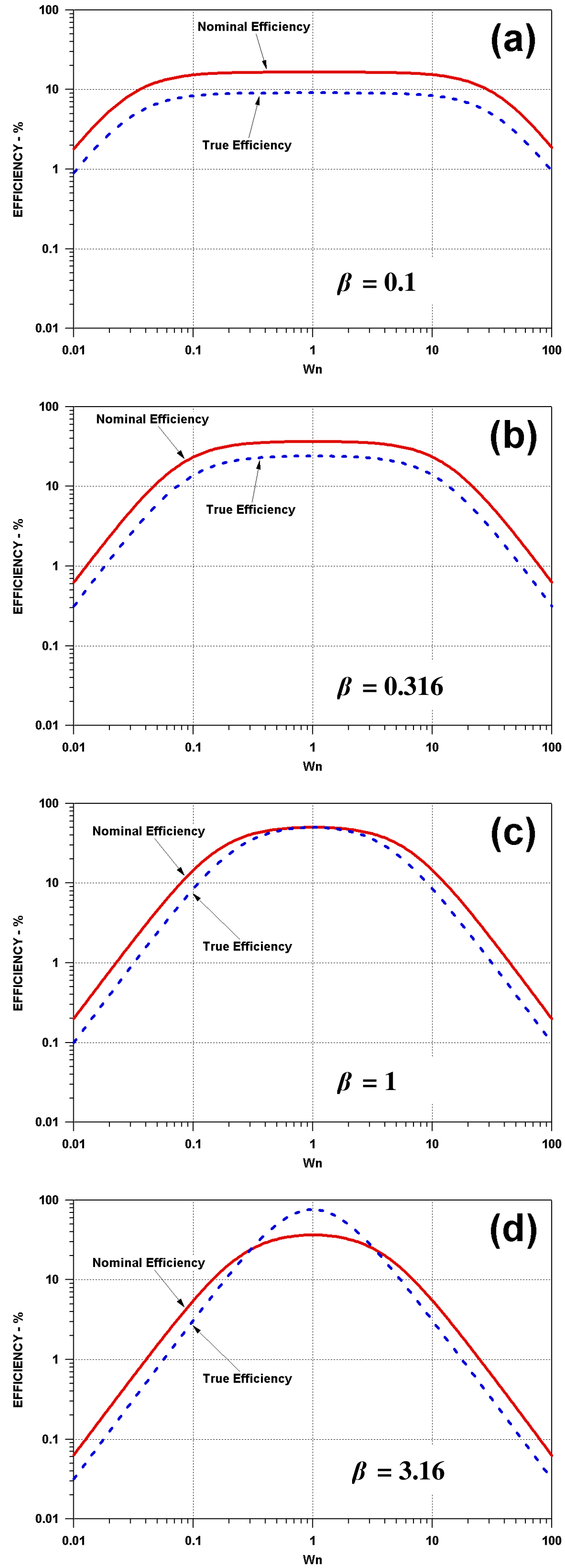



Fig. 12. Graphs showing the nominal and true efficiency frequency responses calculated from the circuit of Fig. 11 for various values of bandwidth $\omega_{MN}$, the moving-mass corner frequency. Driver mechanical losses are neglected ( $\tau = \infty$ ) and $\beta$ = 1. The frequency responses are plotted against the normalized frequency variable $\omega_N$, which is the ratio between frequency and the driver's resonance frequency ( $\omega_n = \omega / \omega_0$ ). (a) $\omega_{MN}$ = 0.316. (b) $\omega_{MN}$ = 1. (c) $\omega_{MN}$ =3.16. (d) $\omega_{MN}$ =10. See text for comments.

*6.4.4.2. Vary Reflected Acoustic Load Resistance ( $\beta$ )*

Figure 13 shows efficiency frequency responses of the circuit Fig. 11 while varying the driver's reflected acoustic load over a two decade range in half-decade steps ( $\beta =$ 0.1, 0.316, 1, 3.16, and 10 ). Here $\omega_{MN} = 3.16$, and $\tau = \infty$ .

Note that as $\beta$ is increased the bandwidth of both the nominal and true responses gets narrower. Note also that the true efficiency is less than the nominal efficiency for $\beta < 1$ and greater for $\beta > 1$ .



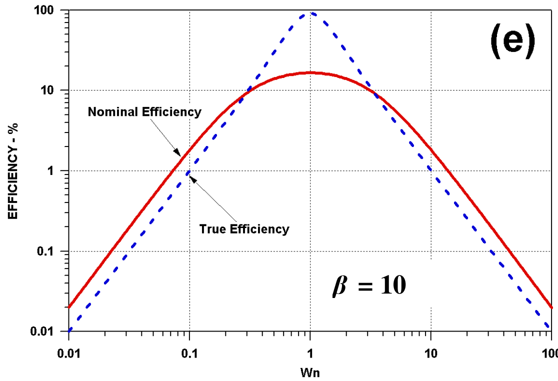
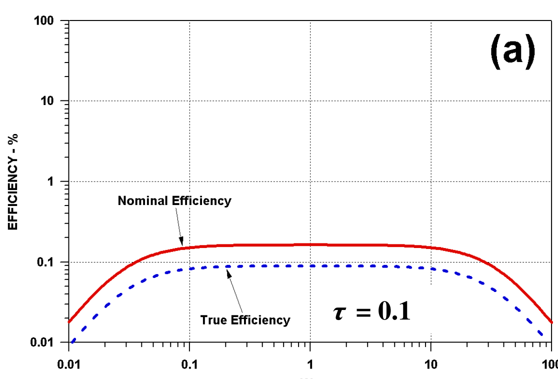



Fig. 13. Graphs showing the nominal and true efficiency frequency responses calculated from the circuit of Fig. 11 for various values of $\beta$. Driver mechanical losses are neglected ($\tau = \infty$). The frequency responses are plotted against the normalized frequency variable $\omega_N$, which is the ratio between frequency and the driver's resonance frequency ($\omega_n = \omega / \omega_0$). $\omega_{MN}$ is held constant at 3.16. (a) $\beta$ = 0.1. (b) $\beta$ = 0.316. (c) $\beta$ = 1. (d) $\beta$ = 3.16. (e) $\beta$ = 10. See text for comments.

*6.4.4.3. Vary Driver Losses ( $\tau$ )*

Figure 14 shows efficiency frequency responses of the circuit Fig. 11 while varying the driver's mechanic losses over a two decade range in half-decade steps ($\tau =$ 0.1, 0.316, 1, 3.16, and 10). Here $\omega_{MN} = 3.16$, and $\beta = 10$. Because the chosen $\beta$ is fairly high ($\beta = 10$), all the responses vary strongly with $\tau$. Note that as $\tau$ increases, the mid-band efficiency increases and the bandwidth decreases. For $\tau > 1$, note that the true efficiency exceeds the nominal efficiency near the resonance frequency of the driver ($\omega_N = 1$).



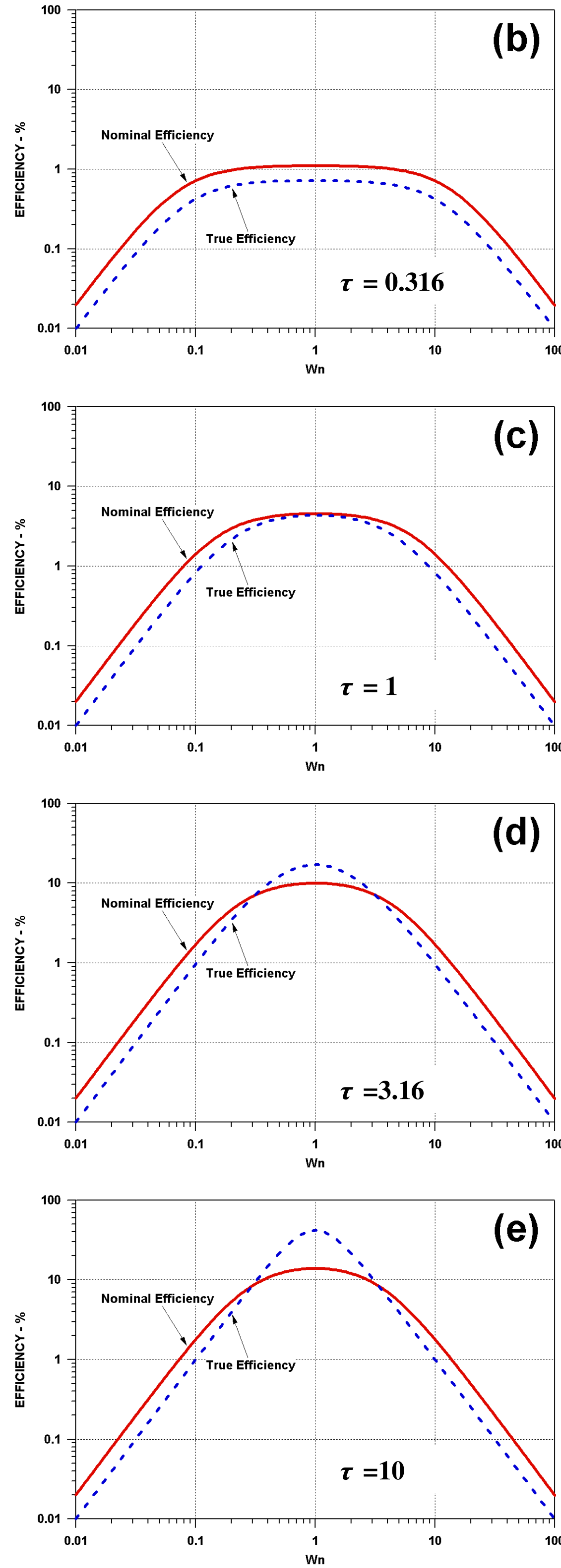




Fig. 14. Graphs showing the nominal and true efficiency frequency responses calculated from the circuit of Fig. 11 for various values of $\tau$. The frequency responses are plotted against the

normalized frequency variable $\omega_N$, which is the ratio between frequency and the driver's resonance frequency ($\omega_n = \omega/\omega_0$). For these graphs $\omega_{MN} = 3.16$, and $\beta = 10$. (a) $\tau$ = 0.1. (b) $\tau$ = 0.316. (c) $\tau$ = 1. (d) $\tau$ = 3.16. (e) $\tau$ = 10. See text for comments.

### *6.4.5. Efficiency-Bandwidth Product*

In this section, the efficiency-bandwidth product of the circuit of Fig. 11 is calculated and displayed in several graphs.

#### *6.4.5.1. Nominal Efficiency*

The maximum true efficiency with driver losses is given by Eq. (16). This maximum occurs at driver resonance ($\omega_N = 1$).

The nominal geometric bandwidth is given by (i.e., if the upper rolloff half-power frequency is 10, then the lower rolloff is 1/10, and the geometric bandwidth is $10^2 = 100$):

$\omega =$

```
> NominalBandwidth;
```

$$\frac{1}{4\tau^2\beta^2}\Big(\tau\,Wmn + Wmn\,\tau\,\beta + Wmn\,\beta + \mathrm{sqrt}\big(Wmn^2\tau^2\beta^2 + Wmn^2\beta^2 + Wmn^2\tau^2 + 2\,Wmn^2\tau^2\beta + 2\,Wmn^2\tau\,\beta + 2\,Wmn^2\beta^2\tau + 4\tau^2\beta^2\big)\Big)\text{^}2 \quad (44)$$

The EB product is then given by the product of Eq. (16) and (44)

$EB_{nom} =$

```
> NominalEfficiencyBandwidthProduct;
```

$$\frac{1}{2\beta(\tau\beta+\tau+\beta)^2}\Big(\tau\,Wmn + Wmn\,\tau\,\beta + Wmn\,\beta + \mathrm{sqrt}\big(Wmn^2\tau^2\beta^2 + Wmn^2\beta^2 + Wmn^2\tau^2 + 2\,Wmn^2\tau^2\beta + 2\,Wmn^2\tau\,\beta + 2\,Wmn^2\beta^2\tau + 4\tau^2\beta^2\big)\Big)\text{^}2 \quad (45)$$

#### *6.4.5.2. True Efficiency*

The maximum true efficiency with driver losses is Eq. (20). This maximum occurs at driver resonance ($\omega_N = 1$).

The geometric bandwidth is given by:

$\omega =$

```
> TrueBandwidth;
```

$$\frac{1}{2\tau^2\beta^2}\Big(Wmn^2\beta^2\tau + Wmn^2\tau^2 + 2\,Wmn^2\tau\,\beta + Wmn^2\tau^2\beta + Wmn^2\beta^2 + 2\tau^2\beta^2 + \mathrm{sqrt}\big(Wmn^4\tau^4 + Wmn^4\beta^4 + 4\,Wmn^2\beta^4\tau^3 + 2\,Wmn^4\beta^4\tau + 4\,Wmn^4\tau^3\beta + 2\,Wmn^4\tau^4\beta + 6\,Wmn^4\tau^2\beta^2 + 4\,Wmn^4\tau\,\beta^3 + Wmn^4\tau^4\beta^2 + 4\,Wmn^2\tau^4\beta^2 + 8\,Wmn^2\tau^3\beta^3 + 4\,Wmn^2\tau^4\beta^3 + 4\,Wmn^2\beta^4\tau^2 + Wmn^4\beta^4\tau^2 + 6\,Wmn^4\beta^2\tau^3 + 6\,Wmn^4\beta^3\tau^2 + 2\,Wmn^4\beta^3\tau^3\big)\Big) \quad (46)$$

The EB product is then given by the product of Eq. (20) and (46)

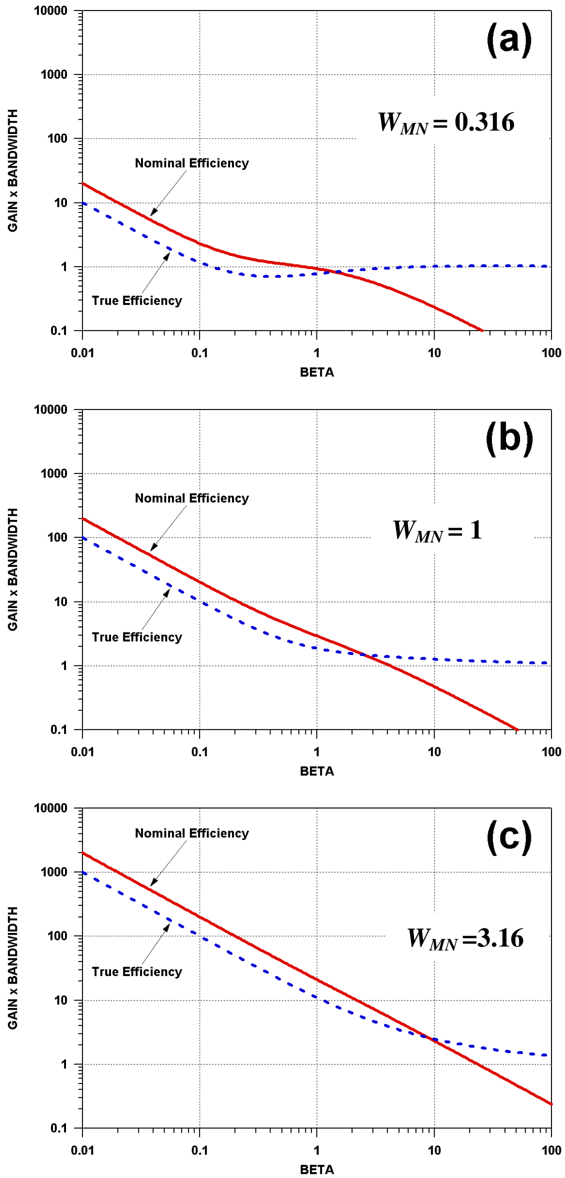
$EB_{true} =$

```
> TrueEfficiencyBandwidthProduct;
```

$$\frac{1}{2\beta(\tau+\beta)(\tau\beta+\tau+\beta)}\Big(Wmn^2\beta^2\tau + Wmn^2\tau^2 + 2\,Wmn^2\tau\,\beta + Wmn^2\tau^2\beta + Wmn^2\beta^2 + 2\tau^2\beta^2 + \mathrm{sqrt}\big(Wmn^2(\tau+\beta)(\tau\beta+\tau+\beta)\big(4\tau^2\beta^2 + Wmn^2\beta^2\tau + Wmn^2\beta^2 + Wmn^2\tau^2\beta + 2\,Wmn^2\tau\,\beta + Wmn^2\tau^2\big)\big)\Big) \quad (47)$$

#### *6.4.5.3. Vary Moving Mass Rolloff Frequency( $\omega_{MN}$ )*

Figure 15 shows plots of the efficiency-bandwidth product plotted versus $\beta$ while varying $\omega_{mn}$ using Eqs. (45) and (47). Note that the EB product increases directly with $\omega_{mn}$. Note also that all the plots indicate that the EB product increases for decreasing $\beta$. At high values of $\beta$, the true EB product exceeds the nominal EB product.



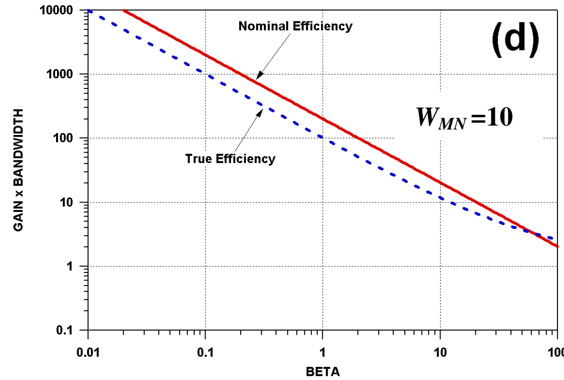
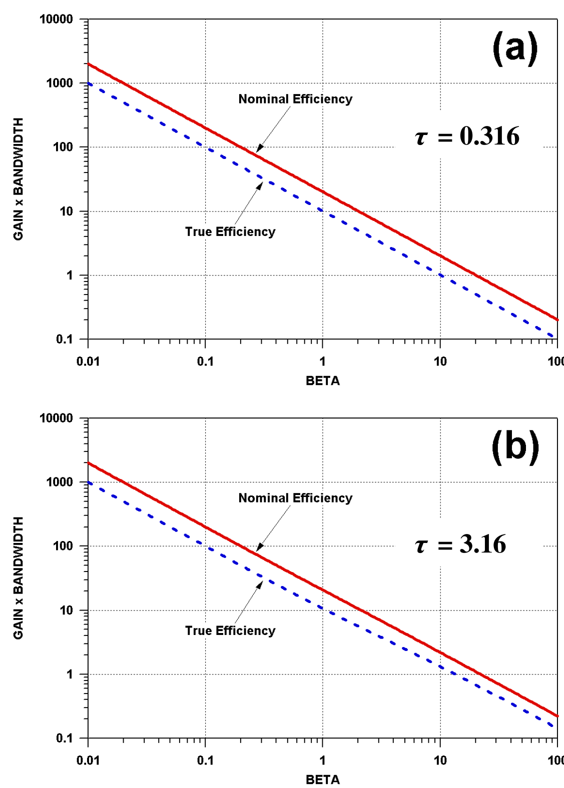




Fig. 15. Plots of the efficiency-bandwidth product of the circuit of Fig. 11 as a function of $\beta$ with bandwidth $\omega_{MN}$ varying in half-decade steps. Driver mechanical losses are neglected ( $\tau = \infty$ ). (a) $\omega_{MN}$ = 0.316. (b) $\omega_{MN}$ = 1. (c) $\omega_{MN}$ = 3.16. (d) $\omega_{MN}$ = 10. See text for comments.

*6.4.5.4. Vary Driver Losses ( $\tau$ )*

Figure 16 shows plots of the efficiency-bandwidth product plotted versus $\beta$ while varying driver mechanical losses ( $\tau$ ) using Eqs. (45) and (47). Note that varying $\tau$ only effects the EB product mainly for high $\beta$ ( $\beta > 10$ ). Note also that in general, the nominal EB product exceeds the true EB product, except for high $\beta$ and high $\tau$ .



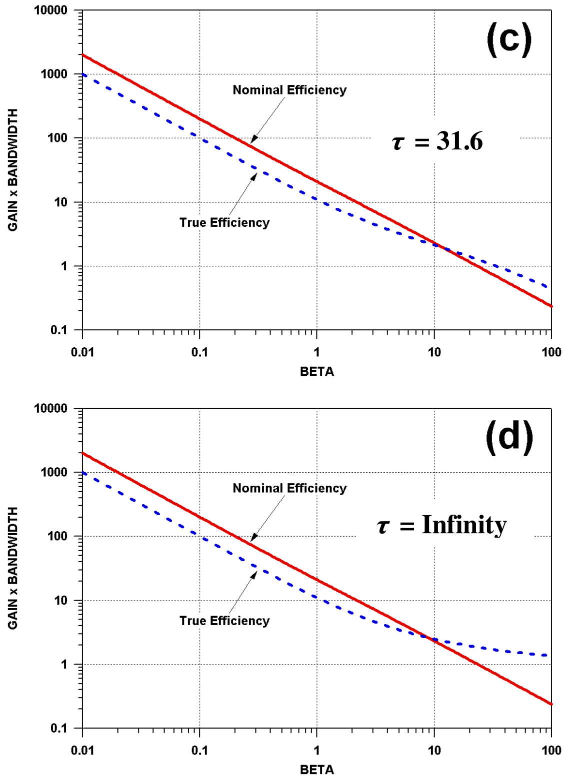



Fig. 16. Plots of the efficiency-bandwidth product of the circuit of Fig. 11 as a function of $\beta$ with mechanical loss $\tau$ varying in decade steps (low losses correspond to high $\tau$). $\omega_{MN}$ = 3.16. (a) $\tau$ = 0.316. (b) $\tau$ = 3.16. (c) $\tau$ =31.6. (d) $\tau$ = $\infty$. See text for comments.

## 6.5. Efficiency: Include All Components

This section analyzes the complete circuit of Fig. 3 and is repeated here as Fig. 12.

Note at paper submission time: Due to time deadlines this section was not completed! Keele, Aug. 25, 2004.

### 6.5.1. *Circuit*

Figure 12 shows the simplified lumped electrical equivalent circuit of the compression driver driving an infinite tube with all components included.

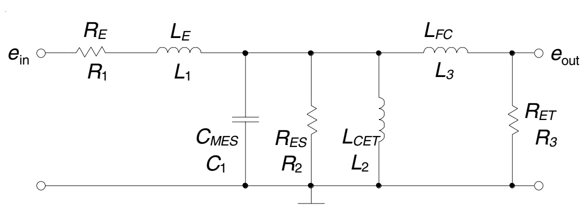



Fig. 14. The complete analogous circuit of Fig. 3 repeated here for reference.

### 6.5.2. *Nominal Efficiency*

The following is raw Maple output for the nominal efficiency as a function of omega in terms of the circuit variables *R1*, *R2*, *R3*, *L1*, *L2*, *L3*, and *C1*. Note that variable subscripting has been suppressed. The denominator of this expression is 8th order in $\omega$.

```
> NominalEfficiencyCollected;
```

$$
\begin{aligned}
&(L2^2\,\omega^2\,R2^2\,R3\,R1)/(C1^2\,R2^2\,L2^2\,L1^2\,L3^2\,\omega^8\\
&+ (L2^2\,L1^2\,L3^2 + C1^2\,R2^2\,L2^2\,R1^2\,L3^2\\
&- 2\,C1\,R2^2\,L2^2\,L1\,L3^2\\
&- 2\,C1\,R2^2\,L2\,L1^2\,L3^2\\
&- 2\,C1\,R2^2\,L2^2\,L1^2\,L3\\
&+ R3^2\,C1^2\,R2^2\,L2^2\,L1^2)\,\omega^6\\
&+ (L1^2\,L2^2\,R2^2 + R3^2\,C1^2\,R2^2\,L2^2\,R1^2\\
&+ R2^2\,L2^2\,L3^2 + R3^2\,L2^2\,L1^2\\
&- 2\,R3^2\,L2^2\,L1\,R2^2\,C1 + 2\,R3\,R2\,L2^2\,L1^2\\
&+ 2\,R2^2\,L2^2\,L1\,L3 + 2\,R2^2\,L2\,L1\,L3^2\\
&+ 2\,R2^2\,L2\,L1^2\,L3 + 2\,R1\,L3^2\,L2^2\,R2\\
&- 2\,C1\,R2^2\,L2^2\,R1^2\,L3 + L2^2\,R1^2\,L3^2\\
&+ R2^2\,L1^2\,L3^2 - 2\,R3^2\,C1\,R2^2\,L2\,L1^2\\
&- 2\,C1\,R2^2\,L2\,R1^2\,L3^2)\,\omega^4\\
&+ (R3^2\,R2^2\,L1^2 + 2\,R3^2\,L2\,L1\,R2^2\\
&+ L2^2\,R3^2\,R2^2 + 2\,R3^2\,R2\,R1\,L2^2\\
&+ R1^2\,R2^2\,L2^2 + 2\,R3\,R2\,L2^2\,R1^2\\
&+ R2^2\,R1^2\,L3^2 + R3^2\,L2^2\,R1^2\\
&+ 2\,R3\,R1\,L2^2\,R2^2 - 2\,R3^2\,C1\,R2^2\,L2\,R1^2\\
&+ 2\,R2^2\,L2\,R1^2\,L3)\,\omega^2 + R3^2\,R2^2\,R1^2)
\end{aligned}
$$

### 6.5.3. *True Efficiency*

The following is raw Maple output for the true efficiency as a function of omega in terms of the circuit variables *R1*, *R2*, *R3*, *L1*, *L2*, *L3*, and *C1*. Note that subscripts have been suppressed. The expression for true efficiency is simpler than the nominal efficiency expression (Section 6.5.2) because the true efficiency is not a function of the driver's voice-coil inductance *L1*. The denominator of this expression is only 6th order in $\omega$.

```
> TrueEfficiencyCollected;
```

$$\left(R2^2\,\omega^2\,L2^2\,R3\right) \Big/ \Big(R1\,R2^2\,\omega^6\,L2^2\,C1^2\,L3^2 + (R1\,L3^2\,L2^2 + R3^2\,R1\,R2^2\,L2^2\,C1^2 + L2^2\,R2\,L3^2 - 2\,R1\,R2^2\,L2\,C1\,L3^2 - 2\,R1\,R2^2\,L2^2\,C1\,L3)\,\omega^4 + (R3^2\,R1\,L2^2 + R3^2\,L2^2\,R2 - 2\,R3^2\,R1\,R2^2\,L2\,C1 + R1\,L2^2\,R2^2 + 2\,R3\,R2\,R1\,L2^2 + R3\,L2^2\,R2^2 + R1\,R2^2\,L3^2 + 2\,R1\,R2^2\,L2\,L3)\,\omega^2 + R3^2\,R1\,R2^2\Big)$$

## 7. VARIATION of PARAMETERS for a TYPICAL DIRECT-RADIATOR HORN DRIVER

This section investigates changing various parameters of an example direct-radiator cone driver used as a compression driver with an infinite tube load. Nominal and true efficiency frequency responses are displayed for various parameter variations including Bl product, moving mass, compression ratio, and driver mechanical losses.

### 7.1. Driver Parameters and Network Model Values

The driver chosen for this example is a five-inch (127 mm) diameter cone driver which has the following mechanical parameters and electric-network values. Driver voice-coil inductance is neglected. The rear cavity volume is assumed infinite (total system compliance is that of the driver only). Front cavity volume is assumed zero. The driver's effective cone diameter is 3.75 inches (47.6 mm) which is also the diameter of the infinite tube load for all the curves labeled "Stock."

#### *7.1.1. Mechanical Parameters*

$M_{MS}$ = 0.0084 kg

$C_{MS}$ = 1.6 x $10^{-4}$ m/N

$S_D$ = 7.13 x $10^{-3}$ $m^2$

$Bl$ = 7.3 N/A

$R_E$ = 7.13 Ohms

$R_{MS}$ = 2.3 $N \cdot \frac{s}{m}$

$V_B$ = Infinite

$f_S$ = 137 Hz

#### *7.1.2. Electric Network Values*

$R_1 = R_E$ = 7.13 Ohms

$R_2 = R_{ES}$ = 23.1 Ohms

$R_3 = R_{ET}$ = 18.0 Ohms

$C_1 = C_{MES}$ = 158 uF

$L_1 = L_E = 0$

$L_2 = L_{CET}$ = 8.53 mH

$L_3 = L_{FC} = 0$

#### *7.1.3. Maximum Efficiencies*

With the electric network values of section 7.1.2, the normalization parameters $\beta$ and $\tau$ can be calculated: $\beta = \frac{R_3}{R_1} = \frac{18.0}{7.13} \approx 2.52$, and $\tau = \frac{R_2}{R_1} = \frac{23.1}{7.13} \approx 3.24$.

With these parameters, the nominal and true peak maximum efficiencies can be calculated using Eqs. (16) and (20). This results in the following efficiencies: $\mathrm{Max}(\eta_{nom}) \approx 0.27$ (27%) and $\mathrm{Max}(\eta_{true}) \approx 0.33$ (33%). These maximum efficiencies occur at the driver's resonance of 137 Hz and are evident in the following graphs for the curves labeled "Stock."

### 7.2. Vary Bl Product

This section illustrates how the efficiency changes as a function of frequency when the driver's *Bl* product is varied.

#### *7.2.1. Nominal Efficiency*

Figure 17 shows the nominal efficiency variations with driver *Bl*.

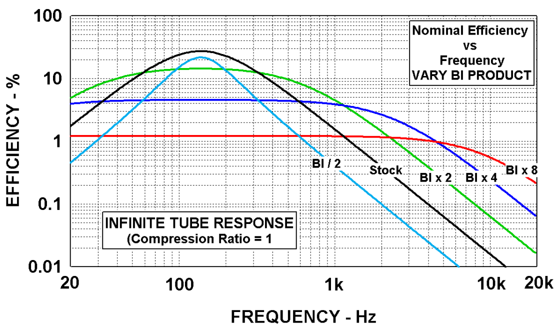



Fig. 17. Nominal efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's *Bl* product varied in octave steps from one-half to eight-times the stock *Bl*. Note that the stock value of *Bl* is near the value that maximizes the nominal efficiency near 137 Hz. Raising or lowering *Bl* reduces the efficiency. Note also that the bandwidth is extended dramatically as *Bl* is raised but the peak efficiency is reduced.

### *7.2.2. True Efficiency*

Figure 18 shows the true efficiency variations with driver *Bl*.

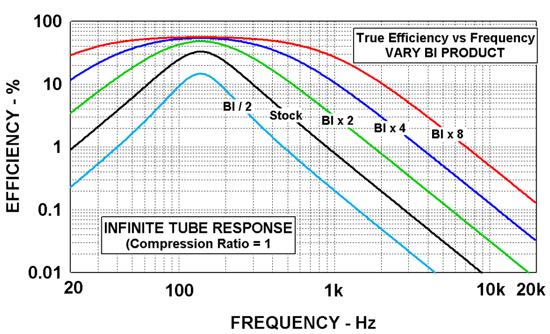



Fig. 18. True efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's *Bl* product varied in octave steps from one-half to eight-times the stock *Bl*. Note that raising the Bl both increases true efficiency and widens the bandwidth.

## 7.3. Vary Moving Mass

This section illustrates how the efficiency changes as a function of frequency when the driver's moving mass is varied.

### *7.3.1. Nominal Efficiency*

Figure 19 shows the nominal efficiency variations with driver moving mass.

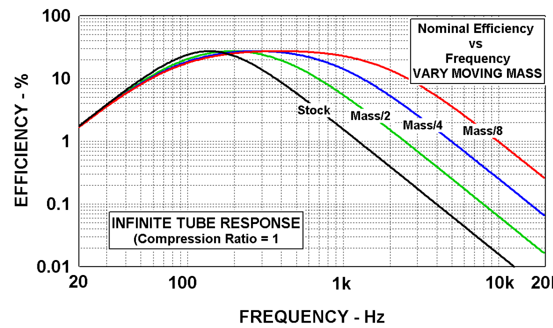



Fig. 19. Nominal efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's moving mass decreased in octave steps from stock to one-eighth stock. Note that maximum efficiency does not change as mass is lowered, but that the high-frequency response is greatly extended.

### *7.3.2. True Efficiency*

Figure 20 shows the true efficiency variations with driver moving mass.

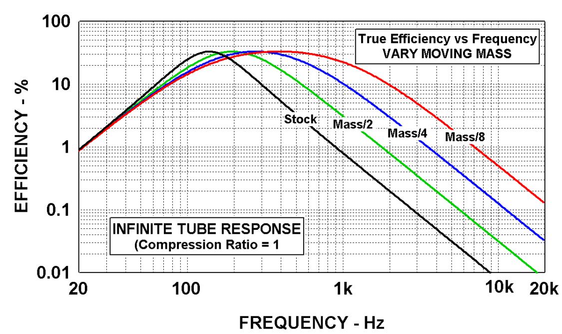



Fig. 20. True efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's moving mass decreased in octave steps from stock to one-eighth stock. Note that maximum efficiency does not change as mass is lowered, but that the high-frequency response is greatly extended. Note also that the peak true efficiency is slightly higher than nominal efficiency shown in Fig. 19.

## 7.4. Vary Compression Ratio

This section illustrates how the efficiency changes as a function of frequency when the system's compression ratio is varied. Note that raising the compression ratio requires that the diameter of the infinite tube be reduced in inverse proportion to the square root of the compression ratio.

### *7.4.1. Nominal Efficiency*

Figure 21 shows the nominal efficiency variations with system compression ratio.

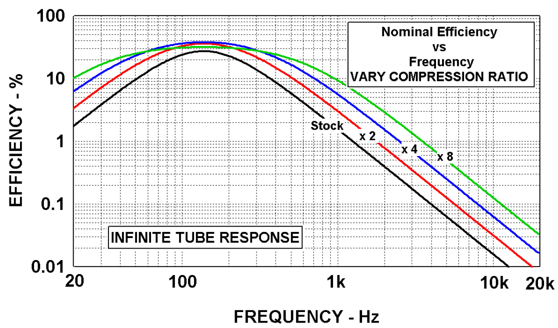



Fig. 21. Nominal efficiency frequency responses of the driver-tube combination of section 7.1 with the driver-tube's compression ratio increased in octave steps from unity to eight-times unity. Note that peak nominal efficiency hardly changes but bandwidth increases as the compression ratio is increased.

### *7.4.2. True Efficiency*

Figure 22 shows the true efficiency variations with system compression ratio.

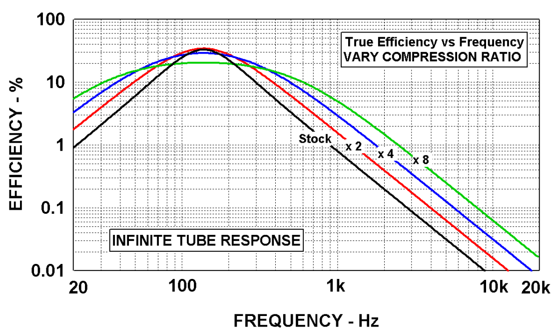



Fig. 22. True efficiency frequency responses of the driver-tube combination of section 7.1 with the driver-tube's compression ratio increased in octave steps from unity to eight-times unity. Note that peak true efficiency reduces somewhat as the compression ratio is increased, but bandwidth increases.

## 7.5. Vary Driver Mechanical Losses

This section illustrates how the efficiency changes as a function of frequency when the driver's mechanical losses are varied.

### *7.5.1. Nominal Efficiency*

Figure 23 shows the nominal efficiency variations with driver mechanical loss.

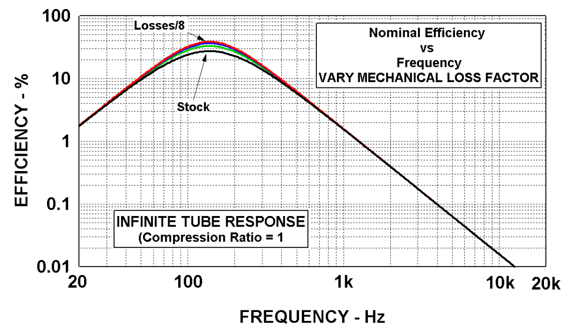



Fig. 23. Nominal efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's mechanical losses reduced in octave steps from stock to one-eighth stock in four steps. Note that the loss reduction only slightly effects the response in a two-octave region around the driver's 137-Hz resonance frequency. Maximum nominal efficiency is only slightly increased as losses are reduced.

### *7.5.2. True Efficiency*

Figure 24 shows the true efficiency variations with driver mechanical loss.

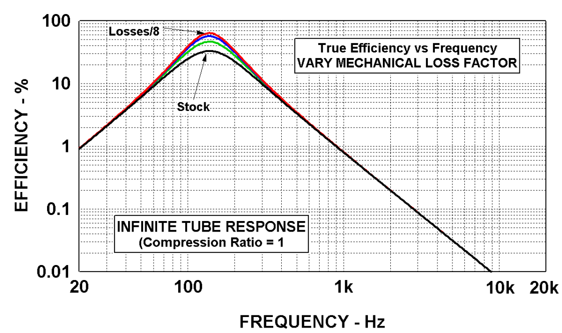



Fig. 24. True efficiency frequency responses of the driver-tube combination of section 7.1 with the driver's mechanical losses reduced in octave steps from stock to one-eighth stock in four steps. As with the nominal efficiency of Fig. 19, loss reduction only effects the response in a region around the driver's 137-Hz resonance frequency. Although the affected region is narrower than the nominal effects shown in Fig. 19, the true efficiency increases more dramatically with lower losses.

## 8. CONCLUSIONS

This paper has investigated the implications of two different definitions of efficiency as it pertains to compression drivers: nominal efficiency and true efficiency. Both definitions compare acoustic output power with input power, but differ in the definition of the input power. The input power for nominal efficiency is calculated by computing the power developed in a fictitious input resistance equal to twice the voice-coil resistance of the compression driver. The input power for true efficiency is the actual input power.

Both definitions of efficiency have a major impact on certain design and performance characteristics of the compression driver including the maximum efficiency attainable, upper and lower bandwidth limits, efficiency-bandwidth products, and selection of compression driver parameters such as Bl product, moving mass, cone area, horn throat area, etc.

This paper has analyzed several simplifications of the compression driver's electrical equivalent circuit to determine maximum efficiency limits, parameter interrelations, and performance limitations. The absolute maximum nominal efficiency was found to be 50% while true efficiency can increase to 100%.

With nominal efficiency, traditional design methods mostly lock in a specific set of driver parameters that maximize efficiency. Designing for maximum true efficiency allows a broader range of driver and system parameters to be used in the design. It was found that the driver's *Bl* product can essentially be increased without limit to maximize the drivers true efficiency and extend the operating range of the driver.

When driver mechanical losses are included, both efficiencies are significantly reduced. The addition of mechanical losses also modifies the true efficiency frequency response so that as in the nominal case, only one set of driver parameters maximizes true efficiency.

The driver's broad-band true efficiency and bandwidth can be increased by raising the Bl force factor as much as possible, while jointly reducing moving mass, voice-coil inductance, mechanical losses, and front air-chamber volume. Increased upper frequency efficiency can be attained by increasing the compression ratio but at the expense of mid-band efficiency.

Maximum efficiency for both the nominal and true definitions occurs at the compression driver's resonance frequency. This implies that the driver's resonance should be placed in the middle of the systems operating band.

Calculations of efficiency-bandwidth product (EBP) has shown that the true efficiency has a significantly higher value of EBP when the *Bl* is raised to high levels. However in general, lowering *Bl* results in higher EBPs, but exhibits lower efficiency.

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